CBSE
MATHEMATICS (Standard), Class-X
SAMPLE QUESTION PAPER
For 2020 Examination
SOLUTIONS WITH MARKING SCHEME

Section ‘A’

1. (c) 3 decimal places

Detailed Answer:
\[\frac{11}{2^3\times5} = \frac{11\times5^2}{(2\times5)^3} = \frac{275}{1000} = 0.275\]

\[\therefore \text{The decimal representation of } \frac{11}{2^3\times5} \text{ will terminate after 3 decimal places.}\]

2. (a) 165

Detailed Answer:
\[N = 60, \ \frac{N}{2} = 30\]

\[\therefore \text{Median class is 160 – 165} \]

[\[\therefore \text{160 – 165, class interval contain 30 as cumulative frequency}\]

\[\therefore \text{Upper limit of C.I. 160 – 165 is 165.}\]

3. (c) 20

Detailed Answer:
Smallest two digit composite number = 10
Smallest composite number = 4
\[\therefore \text{LCM (10, 4) = 2 \times 5 \times 2 = 20}\]

4. (a) all real values except 10

Detailed Answer:
\[\therefore a_1 = 3, b_1 = -1, c_1 = -5\]
\[a_2 = 6, b_2 = -2, c_2 = -p\]

\[\therefore \text{The given pair of lines will be parallel if } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\]

\[\frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{-p} \Rightarrow \frac{1}{2} \neq \frac{5}{p}\]

\[p \neq 10\]

\[\therefore \text{The answer is all real value except 10.}\]

5. (d) not defined

Detailed Answer:
In \(\triangle ABC, \angle C = 90^\circ\)
\[\therefore C = A + B\]

[By angle sum property of triangle]

\[\therefore A + B = 90^\circ\]

\[\sec (A + B) = \sec 90^\circ = \text{not defined.}\]

6. (a) \(\sqrt{2} - 1\)

Detailed Answer:
\[\sin\theta + \cos\theta = \sqrt{2} \cos\theta\]
\[\tan\theta + 1 = \sqrt{2} \quad \text{[Divide by } \cos\theta \text{ on both side]}\]
\[\tan\theta = \sqrt{2} - 1\]

7. (d) 30°

Detailed Answer:
Given: \(\sin\alpha = \frac{\sqrt{3}}{2}, \cos\beta = 0\)

If \(\sin\alpha = \sin 60^\circ\) \(\Rightarrow \alpha = 60^\circ\)

If \(\cos\beta = 0 = \cos 90^\circ\) \(\Rightarrow \beta = 90^\circ\)

\[\therefore \beta - \alpha = 90^\circ - 60^\circ = 30^\circ\]

8. (d) IV quadrant

Detailed Answer:
Let \(P(x, y)\) be the point.
\[\therefore m_1 : m_2 = 1 : 2, (x_1, y_1) = (8, -9) \quad (x_2, y_2) = (2, 3)\]

\[\therefore x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1 \times 2 + 2 \times 8}{1 + 2} = \frac{18}{3} = 6\]
y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1 \times 3 + 2 \times -9}{1 + 2} = \frac{3 - 18}{3} = -5

\therefore \text{Comparing the given equation with } ax^2 + bx + c = 0
a = (k - 1), b = -10, c = 3
\therefore \frac{1}{\alpha} = \frac{3}{(k - 1)} \Rightarrow 1 = \frac{3}{k - 1} \Rightarrow k = 4

\text{OR}

9. (c) 4

\text{Detailed Answer :}
The distance of the point P(–3, –4) from X-axis (in units) is 4.

\begin{align*}
\text{10. (a) } & -12 \\
\text{Detailed Answer :} & \quad A\left(\frac{m}{3}, 5\right) \text{ is the mid point of the line joining} \\
& \quad \text{the points } Q(-6, 7) \text{ and } R(-2, 3). \\
& \therefore m = \frac{(-6) + (-2)}{2} \Rightarrow m = -4 \times 3 \\
& \therefore m = -12
\end{align*}

\begin{align*}
\text{11. } & \pi rl + 2\pi rh + \pi r^2 \\
\text{ Detailed Answer :} & \quad \text{The total surface area of the given solid figure} \\
& \quad \text{= curved surface area of cone + curved} \\
& \quad \text{surface area of cylinder + surface area of} \\
& \quad \text{circular base} \\
& \quad = \pi rl + 2\pi rh + \pi r^2
\end{align*}

\begin{align*}
\text{12. } & 4 \\
\text{ Detailed Answer :} & \quad \text{Let one root of given quadratic equation be } \alpha. \\
& \text{other root be } \frac{1}{\alpha}. \quad \therefore \text{For a quadratic equation :} \\
& \text{Product of roots } = \frac{c}{a}
\end{align*}

\begin{align*}
\text{13. } & \frac{49}{81} \\
\text{ Detailed Answer :} & \quad \text{Here number of zeroes } = \text{ the no. of intersection} \\
& \quad \text{made by polynomial } p(x) \text{ on x-axis } = 5.
\end{align*}

\begin{align*}
\text{14. } & 14, 38 \\
\text{ Detailed Answer :} & \quad \text{Let } b’ \text{ and } c’ \text{ be the second and forth term of} \\
& \quad \text{given sequence} \\
& \therefore b = \frac{26 + 2}{2} = 14 \\
& \therefore c = 26 + 12 = 38
\end{align*}

\begin{align*}
\text{15. } & \frac{3}{11} \\
\text{ Detailed Answer :} & \quad \text{Total number of outcomes } (E) = 11 \\
& \text{Favourable number of outcomes } (F) = 3 (-1, 0, 1) \\
& \therefore \text{Probability } P(F) = \frac{\frac{3}{11}}{\frac{3}{11}} = 1
\end{align*}

\begin{align*}
\text{16. Rational number } = & 0.30 \frac{1}{2} \\
\text{Irrational number } = & 0.3010203040… \frac{1}{2} \\
\text{Or any other correct rational and irrational number}
\end{align*}
17. $\triangle ACB \sim \triangle ADC$ (AA criterion) \\
$\Rightarrow \frac{AC}{AD} = \frac{AB}{AC}$ \\
$\therefore AB = 12 \text{ cm}$ 

[CBSE Marking Scheme, 2020]

Detailed Answer:
Given: $\angle ACB = \angle CDA$, 
$AC = 6 \text{ cm}, AD = 3 \text{ cm}$
In $\triangle ADC$ and $\triangle ACB$
(i) $\angle CAD = \angle CAB$ (common)
(ii) $\angle ADC = \angle ACB$ (given)
$\therefore \triangle ADC \sim \triangle ACB$ (A A criterion)
So,
$\frac{AC}{AD} = \frac{AB}{AC}$
or,
$AB = 12 \text{ cm}$

18. $\text{In } \triangle OBP, \frac{OB}{OP} = \sin 30^\circ$ \\
$\therefore OP = 2r$ 

[CBSE Marking Scheme, 2020]

OR

Length of Tangent = $2 \times \sqrt{5^2 - 4^2} = 2 \times 3 \text{ cm} = 6 \text{ cm}$ 

[CBSE Marking Scheme, 2020]

Detailed Answer:
In the adjacent figure
$OD = 4 \text{ cm}$ (radius of inner circle) 
$OB = 5 \text{ cm}$ (radius of outer circle)
Now, $\angle ODB, \angle D = 90^\circ$
Apply Pythagoras theorem,
$OB^2 = OD^2 + BD^2$
or,
$BD^2 = OB^2 - OD^2$
$BD = \sqrt{5^2 - 4^2}$ 
$BD = 3 \text{ cm}$

Hence, the length of chord $AB = 2 \times BD$ 
$= 2 \times 3 \text{ cm}$ 
$= 6 \text{ cm}$

19. $b, c$ and $2b$ are in A.P $\Rightarrow c = \frac{3b}{2}$ 
$\therefore b : c = 2 : 3$ 

[CBSE Marking Scheme, 2020]

20. $D = (2\sqrt{2k})^2 - 4(1)(18) = 0 \Rightarrow k = \pm 3$ 

[CBSE Marking Scheme, 2020]

Section ‘B’

21. $110, 120, 130, \ldots, 990$ 
$\Rightarrow a_n = 990 \Rightarrow 110 + (n - 1) \times 10 = 990$ 
$\therefore n = 89$ 

22. $\text{In } \triangle ADE \sim \triangle GBD$ and $\triangle ADE \sim \triangle FCE$ 
$\Rightarrow \triangle GBD \sim \triangle FCE$ (AA Criterion) 
$\Rightarrow \frac{GD}{FC} = \frac{GB}{FE}$ 
$\Rightarrow \frac{GD}{FE} \Rightarrow GB \times FE = GB \times FC = BG \times FC$ 

[CBSE Marking Scheme, 2020]

Detailed Answer:
In $\triangle ADE$ and $\triangle GBD$
(i) $\angle ADE = \angle DBG$ (corresponding Angles) 
(ii) $\angle DAE = \angle BGD = 90^\circ$
$\triangle ADE \sim \triangle GBD$ (By AA criterion) ... (i)
Similarly $\triangle ADE \sim \triangle FEC$ ... (ii)

from equation (i) and (ii)
$\triangle GBD \sim \triangle FEC$

So,
$\frac{GD}{FC} = \frac{GB}{FE}$
or,
$GD \times FE = GB \times FC = GF \times GF = GB \times FC = GF^2 = GB \times FC$
Section 'C'  

27. Let us assume, to the contrary, that \(2\sqrt{5} - 3\) is a rational number.

\[
AD \perp BC
\]

\[
\therefore \text{In } \triangle ABD, \quad AB^2 = AD^2 + BD^2
\]

\[
\Rightarrow \quad AB^2 = AD^2 + \frac{BC^2}{4}
\]

or \[4AB^2 = 4AD^2 + BC^2\]

\[
\Rightarrow \quad 3AB^2 = 4AD^2
\]

\[
\therefore \quad 2\sqrt{5} - 3 = \frac{p}{q}, \text{ where } p \text{ and } q \text{ are integers and } q \neq 0
\]

\[
\Rightarrow \quad \sqrt{5} = \frac{p + 3q}{2q}
\]

(1)

Since \(p\) and \(q\) are integers : \(\frac{p + 3q}{2q}\) is a rational number.

\[
\therefore \quad \sqrt{5} \text{ is a rational number which is a contradiction as } \sqrt{5} \text{ is an irrational number.}
\]

Hence our assumption is wrong and hence \(2\sqrt{5} - 3\) is an irrational number.

OR

180 = 144 \times 1 + 36

\[
144 = 36 \times 4 + 0
\]

\[
\therefore \text{HCF(180, 144) = 36}
\]

So, \(36 = 13m - 16\)

Solving, we get \(m = 4\)

28. \[
S_n = \sum m \left[2a + (m - 1)d\right] = \frac{n}{2} \left[2a + (n - 1)d\right]
\]

\[
\Rightarrow 2a(m - n) + d(m^2 - m - n^2 + n) = 0
\]

or, \(m - n) \left[2a + (m + n - 1)d\right] = 0 \text{ or } S_{m+n} = 0\)

Detailed Answer:

Let \(a\) and \(d\) be the first term and common difference of AP.

According to the problem.

\[
S_m = S_n
\]

\[
\frac{m}{2} \left[2a + (m - 1)d\right] = \frac{n}{2} \left[2a + (n - 1)d\right]
\]

or, \(2a(m - n) + d(m^2 - m - n^2 + n) = 0\)

or, \(m - n) \left[2a + (m + n - 1)d\right] = 0\)

So, either \(m - n = 0\) but \(m \neq n\)

or, \(2a + (m + n - 1)d = 0\) \hspace{1cm} \text{(i)}

Now, \[
S_{m+n} = \frac{m+n}{2} \left[2a + (m + n - 1)d\right]
\]

\[
= \frac{m+n}{2} \times 0
\]

\[
= 0
\]

Hence, \(S_{m+n} = 0\)

29. \(x + y = 7\) and \(2(x - y) + x + y + 5 + 5 = 27\)

\[
\therefore x + y = 7 \text{ and } 3x - y = 17
\]

Solving, we get, \(x = 6\) and \(y = 1\)

OR

\[
\frac{1}{x} = a \text{ and } \frac{1}{y} = b
\]

1
Solutions

\[ 21a + 47b = 110 \text{ and } 47a + 21b = 162 \]

Adding and subtracting the two equations, we get 
\[ a + b = 4 \text{ and } a - b = 2 \]

Solving the above two equations, we get \( a = 3 \) and \( b = 1 \)

\[ \therefore x = \frac{1}{3} \text{ and } y = 1 \]

30. \( p(x) = x^4 + 4x^3 - 2x^2 - 20x - 15 \)
\( x^2 - 5 \) is a factor of \( p(x) \)

\[ \therefore p(x) = (x^2 - 5)(x^2 + 4x + 3) \]

or \( p(x) = (x^2 - 5)(x + 3)(x + 1) \)

So, all the zeroes of \( p(x) \) are \( \sqrt{5}, -\sqrt{5}, -3 \) and \(-1\)

Detailed Answer:
Since, \( (x^2 - 5) \) in a factor of \( p(x) = x^4 + 4x^3 - 2x^2 - 20x - 15 \)

So,
\[ x^2 - 5 \]
\[ \frac{x^4 + 4x^3 - 2x^2 - 20x - 15}{x^2 + 4x + 3} = \frac{+4x^2 + 3x^2 - 20x - 15}{+4x^2 + 3x^2 - 20x - 15} - \frac{+x^2}{+x^2} - \frac{15}{+15} \]

Now again factorising \( x^2 + 4x + 3 \) as
\[ x^2 + 4x + 3 = x^2 + 3x + 1x + 3 \]
\[ = x(x + 3) + 1(x + 3) \]
\[ = (x + 3)((x + 1) \]

So, complete factors of \( p(x) \) are \( (x - \sqrt{5}), (x + \sqrt{5}) \)

\( x + 3 \) and \( x + 1 \)

Hence, all zeroes of \( p(x) \) are, \( \sqrt{5}, -\sqrt{5}, -3 \) and \(-1\)

31. (i) A(1,7), B(4,2) C(-4,4)

Distance travelled by Seema (AC)
\[ = \sqrt{(4 - 1)^2 + (4 - 7)^2} = \sqrt{25 + 9} = \sqrt{34} \text{ units} \]

Distance travelled by Aditya (BC)
\[ = \sqrt{(4 - (-4))^2 + (4 - 7)^2} = \sqrt{64 + 4} = \sqrt{68} \text{ units} \]

\( \therefore \) Aditya travels more distance

(ii) Coordinates of D are \( \left(\frac{1 + 4}{2}, \frac{7 + 2}{2}\right) \)
\[ = \left(\frac{5}{2}, \frac{9}{2}\right) \]

(iii) \( \text{ar} \ (\triangle ABC) = \frac{1}{2} \left[1(2 - 4) + 4(4 - 7) - 4(7 - 2)\right] \)
\[ = 17 \text{ sq. units} \]

32. \( \sin \theta + \cos \theta = \sqrt{3} \Rightarrow (\sin \theta + \cos \theta)^2 = 3 \)
\[ \Rightarrow 1 + 2 \sin \theta \cos \theta = 3 \Rightarrow \sin \theta \cos \theta = 1 \]
\( \therefore \tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 1 = \text{RHS} \)
OR
\[ \cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta) \]
\[ \frac{\tan(60^\circ + \theta) \times \tan(30^\circ - \theta)}{+ (\cot 30^\circ + \sin 90^\circ) \times (\tan 60^\circ - \sec 0^\circ)} \]
\[ \cos^2(45^\circ + \theta) + \sin^2(45^\circ + \theta) \]
\[ = \frac{\tan(60^\circ + \theta) \times \cot(60^\circ + \theta)}{+ (\sqrt{3} + 1) \times (\sqrt{3} - 1)} \]
\[ = 1 + 2 = 3 \]

33. Required Area = Area of triangle - Area of 3 sectors
Area of Triangle = \( \frac{1}{2} \times 24 \times 7 = 84 \text{ m}^2 \)
Area of three sectors = \( \frac{\pi r^2}{360^\circ} \times \text{(sum of three angles of triangle)} \)
\[ = \frac{22 \times 7 \times 180^\circ}{7 \times 2 \times 2 \times 360^\circ} = \frac{77}{4} \text{ or } 19.25 \text{ m}^2 \]
\( \therefore \) Required Area = \( \frac{259}{4} \) or \( 64.75 \text{ m}^2 \)

34. (i) Curve 1 – Less than ogive, Curve 2 – More than ogive
(ii) Median Rainfall = 21 cm
(iii) 3 Median = Mode + 2 mean
\( \therefore \) Mode = 16.2 cm

Section ‘D’

35. Correct construction of given triangle
Correct construction of similar \( \triangle \) with scale factor \( \frac{3}{4} \)

[CBSE Marking Scheme, 2020]

Detailed Answer

Steps of constructions:
1. Draw base BC of side 6.5 cm.
2. Draw \( \angle B = 30^\circ \).
3. Draw \( \angle C = 180^\circ - (105^\circ + 30^\circ) = 45^\circ \).
4. Let point A be the point where the two intersect.
5. Draw a ray BX making an acute angle with BC on the side opposite to vertex A.
6. Mark 4 points on BX such that BB₁ = B₁B₂ = B₂B₃ = B₃B₄.
7. Join B₄ to B.
8. Draw parallel line B₃ B’ parallel to B₄ B.
9. Again draw parallel line B’A’ parallel to BA. ΔA’BC is the required triangle.

36. For correct given, to prove, const. \( \left( 4 \times \frac{1}{2} = 2 \right) \)
and figure
For correct proof \( 2 \)

[CBSE Marking Scheme, 2020]

Detailed Answer

Explanation:
Given: A triangle ABC in which a line DE parallel to side BC intersects other two sides, AB and AC at D and E respectively.

To prove: \( \frac{AD}{DB} = \frac{AE}{EC} \)

Construction: Join BE and CD and then draw DM ⊥ AC and EN ⊥ AB.

Proof: Area of \( \triangle ADE = \left( \frac{1}{2} \times \text{base} \times \text{height} \right) \)

So, \( \text{ar}(\triangle ADE) = \frac{1}{2} \times AD \times EN \)

\( \text{ar}(\triangle BDE) = \frac{1}{2} \times DB \times EN \)

Similarly, \( \text{ar}(\triangle ADE) = \frac{1}{2} \times AE \times DM \)

and \( \text{ar}(\triangle DEC) = \frac{1}{2} \times EC \times DM \)

Therefore,
\[
\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad \ldots \text{(i)}
\]

and
\[
\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DEC)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \ldots \text{(ii)}
\]

Since, \( \triangle BDE \) and \( \triangle DEC \) are on the same base DE and between the same parallel lines BC and DE.

\( \therefore \) ar\( \triangle BDE \) = ar\( \triangle DEC \) \( \ldots \text{(iii)} \)

From (i), (ii), (iii)
We have \( \frac{AD}{DB} = \frac{AE}{EC} \)

Hence Proved

37. Let the original speed of the train be \( x \) km/h

\[
\frac{360}{x} - \frac{360}{x + 5} = \frac{48}{60}
\]

2
⇒ \( x^2 + 5x - 2250 = 0 \)
⇒ \((x + 50)(x - 45) = 0 \therefore x = 45 \)
Hence original speed of the train = 45 km/h

\[
\frac{1}{x} - \frac{1}{x-2} = 3
\]
\[
\frac{x-2-x}{x(x-2)} = \frac{3}{1}
\]
\[
x^2 - 6x = -2
\]
\[
x^2 - 6x + 2 = 0
\]
\[
x = \frac{6 \pm \sqrt{12}}{6}
\]
\[
= \frac{3 + \sqrt{3}}{3}, \frac{3 - \sqrt{3}}{3}
\]

38. Capacity of tank = \( \frac{1}{3} \pi \times 20 \times (10^2 + 25^2 + 10 \times 25)m^3 \)
= \( \pi \times 20 \times 325m^3 = \pi \times 20 \times 325 \) l

\[ \text{Cost of petrol} = \pi \times 20 \times 325 \times 70 = ₹1430000 \]

Slant height = \( \sqrt{20^2 + 25^2} = 25m \)

Surface area of tank = \( \pi \times 25(10 + 25)m^2 = 2750m^2 \)

OR

Quantity of water flowing through pipe in 1 hour
= \( \pi \times \frac{7}{100} \times \frac{7}{100} \times 15000 m^3 \)

Required time = \( \frac{50 \times 44 \times 21}{100} + \left( \pi \times \frac{7}{100} \times \frac{7}{100} \times 15000 \right) \)
= 2 hours

39. Correct figure

In \( \triangle ABE \), \( \frac{BE}{AB} = \tan 60^\circ \)
⇒ \( AB = 3000 m \)

In \( \triangle DAC \), \( \frac{DC}{AC} = \tan 30^\circ \)
⇒ \( AC = 9000 m \)

\[ \text{BC} = AC - AB = 6000m \]

\[ \therefore \text{Speed of aeroplane} = \frac{6000}{30} \text{ m/s} = 200 \text{ m/s} \]

40.

<table>
<thead>
<tr>
<th>Daily Wages (in Rs.)</th>
<th>Number of Workers ( f_i )</th>
<th>( x_i )</th>
<th>( u_i )</th>
<th>( f_i u_i )</th>
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<tr>
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<td>10</td>
<td>110</td>
<td>-3</td>
<td>-30</td>
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<td>120-140</td>
<td>15</td>
<td>130</td>
<td>-2</td>
<td>-30</td>
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<tr>
<td>Total</td>
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</table>

\[ \text{Mean daily wages} = 170 + \frac{1}{110} \times 20 = ₹170.19 \text{ (approx.)} \]

\[ \text{Mode} = 160 + \frac{22 - 20}{44 - 20 - 18} \times 20 = ₹166.67 \text{ (approx.)} \]

Detailed Answer:

\[ \sum f_i = 110 \]
\[ \sum f_i u_i = 1 \]

\[ \text{Mean of daily wages} = A + \frac{\sum f_i u_i \times h}{\sum f_i} \]

(here, \( h = 20 \))

\[ = 170 + \frac{1}{110} \times 20 \]
\[ = ₹ 170.19 \text{ (approx.)} \]
For Mode, we choose class interval (160 – 180) because it has highest frequency 22.
So, $l = 160, f_1 = 22, f_0 = 20, f_2 = 18, h = 20$

Mode ($M_o$) = $l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

= $160 + \left( \frac{22 - 20}{44 - 20 - 18} \right) \times 20$

= ₹ 166.67 (approx)