Strictly Based on Latest Syllabus Issued by CBSE
for 2014 Examination

Sample Question Papers

Solutions

Mathematics

Class 11

OSWAAL BOOKS

CONTENTS

- **Solutions**

- **Sample Question Paper 6** 1 - 11
- **Sample Question Paper 7** 12 - 22
- **Sample Question Paper 8** 23 - 32
- **Sample Question Paper 9** 33 - 45
- **Sample Question Paper 10** 46 - 57

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SAMPLE QUESTION PAPER–6

Section-A

1. We have $R = \{(x, y) : (x - y)$ is odd natural number $x \in A, y \in B\}$. **1**

$\therefore$ $R = \{5, 4\}$

2. $$f(x) = \frac{1}{\sqrt{5-x}}.$$

It is defined when $5 - x > 0 \Rightarrow x < 5$.

$\therefore$ Domain of $f = \{x : x \in R, x < 5\}$

Range $= (0, \infty)$. **1**

3. $$\lim_{x \to 0} \frac{\tan 8x}{\sin 3x} = \lim_{x \to 0} \left[\frac{\left(\frac{\tan 8x}{8x}\right) \times 8x}{\left(\frac{\sin 3x}{3x}\right) \times 3x} \right] = \frac{8}{3}.$$ **1**

4. $$\text{cosec}\,(-1410°) = \text{cosec}\,(-360° \times 4 + 30)$$
$$= \text{cosec}\,30° = 2.$$ **1**

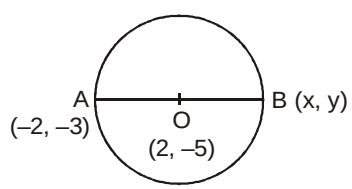
5. $$\frac{x-2}{2} = 2$$

$\Rightarrow$ $x = 4 + 2 = 6$

and $$\frac{y-3}{2} = -5$$

$\Rightarrow$ $y - 3 = -10$

$y = -10 + 3 = -7$



$\therefore$ Co-ordinates of B are $(6, -7)$. **1**

6. If two lines do not intersect in plane, then they are parallel. **1**

Section - B

7. $$\text{L.H.S.} = \frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$$ **1**

$$= \frac{b^2 + c^2 - a^2}{2abc} + \frac{a^2 + c^2 - b^2}{2abc} + \frac{a^2 + b^2 - c^2}{2abc}$$ **2**

$$= \frac{b^2 + c^2 - a^2 + a^2 + c^2 - b^2 + a^2 + b^2 - c^2}{2abc}$$ **1**

$$= \frac{a^2 + b^2 + c^2}{2abc} = \text{R.H.S.}$$ **Hence proved.** **1**

OR

Given that, $\cot \alpha = \frac{1}{2}$ and $\sec \beta = -\frac{5}{3}$ **1**

$\Rightarrow$ $\tan \alpha = 2$ and $\tan \beta = \sqrt{\left(\frac{-5}{3}\right)^2 - 1}$

$$\tan\beta = \sqrt{\frac{25-9}{9}}$$

$$\tan\beta = \pm\frac{4}{3} \qquad [\because B \in \left(\frac{\pi}{2}, \pi\right)]$$ **2**

$$\therefore \quad \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$ **1**

$$= \frac{2-\frac{4}{3}}{1+2+\frac{4}{3}} = \frac{\frac{2}{3}}{\frac{11}{3}}$$ **1**

$$\therefore \quad \tan(\alpha+\beta) = \frac{2}{11}$$

8. L.H.S. $= \cot 7\frac{1}{2}^\circ$

$$= \frac{\cos 7\frac{1}{2}^\circ}{\sin 7\frac{1}{2}^\circ}$$ **1**

$$= \frac{2\cos 7\frac{1}{2}^\circ . \cos 7\frac{1}{2}^\circ}{2\sin 7\frac{1}{2}^\circ . \cos 7\frac{1}{2}^\circ}$$

$$= \frac{2\cos^2 7\frac{1}{2}^\circ}{\sin 15^\circ}$$

$$= \frac{1+\cos 15^\circ}{\sin 15^\circ}$$ **1**

$$= \frac{1+\cos(45^\circ - 30^\circ)}{\sin(45^\circ - 30)^\circ}$$

$$= \frac{1+\cos 45^\circ\cos 30^\circ + \sin 45^\circ \sin 30^\circ}{\sin 45^\circ . \cos 30^\circ - \cos 45^\circ \sin 30^\circ}$$

$$= \frac{1+\left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}\right)}$$ **1**

$$= \frac{\left(1+\frac{(\sqrt{3}+1)}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$= \left(\frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1}\right)$$

$$= \frac{(2\sqrt{2}+\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{2\sqrt{6}+3+\sqrt{3}+2\sqrt{2}+\sqrt{3}+1}{(\sqrt{3})^2-1^2}$$

$$= \frac{2\sqrt{6}+2\sqrt{3}+2\sqrt{2}+4}{2}$$

$$= \sqrt{6}+\sqrt{3}+\sqrt{2}+2$$

$$= \sqrt{6}+\sqrt{3}+\sqrt{2}+\sqrt{4}$$

= R.H.S. **Hence proved.** **1**

9. We have $2kx - 5k = (x - 3).x^2$

$\Rightarrow$ $(k - 3).x^2 - 2kx + 5k = 0$

$\therefore$ $= (-2\,k)^2 - 4\,(k - 3).\,5k$ **1**

$= 4k^2 - 20k\,(k - 3)$

$= 4k\,(k - 5k + 15)$

$= 4k\,(-\,4k + 15)$

$= 4k\,(4k - 15)$ **1**

For real and unequal root

$\Delta > 0$ **1**

$\Rightarrow$ $-\,4k\,(4k - 15) > 0$

$\Rightarrow$ $4k\,(4k - 15) < 0$

$\Rightarrow$ $0 < k < \frac{15}{4}$ **1**

OR

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+i\sqrt{2})-(\sqrt{3}-i\sqrt{2})} = \frac{9+5}{2i\sqrt{2}} \times \frac{\sqrt{2}\,i}{\sqrt{2}\,i}$$ **2**

$$= \frac{7\sqrt{2}\,i}{2i^2} = -\frac{7\sqrt{2}\,i}{2}$$

$$= 0 - \frac{7}{2}\sqrt{2}\,i$$ **2**

10. Veena visits to four cities *A, B, C* and *D.*

Total arrangements in which Veena can visit = 4 ! = 24 **1**

(i) She visits *A* before *B* and *B* before *C*

$\therefore$ $X = \{ABCD, DABC, ABDC, ADBC\}$

$\therefore$ $P(X) = \frac{4}{24} = \frac{1}{6}$ **1**

(ii) Y = Order of cities *A* first and *B* last

$\therefore$ $Y = \{ACDB, ADCB\}$

$P(Y) = \frac{2}{24} = \frac{1}{12}.$ **2**

11. We have a pack of 52 playing cards

Total ways of selecting 4 cards = ${}^{52}C_4 = \frac{52 \times 51 \times 50 \times 49}{4 \times 3 \times 2 \times 1}$ **1**

A = Event of getting 1 card from each suit

$\therefore$ $$P(A) = \frac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{\frac{52 \times 51 \times 50 \times 49}{4 \times 3 \times 2 \times 1}}$$ **1**

$$= \frac{13 \times 13 \times 13 \times 13 \times 4 \times 3 \times 2}{52 \times 51 \times 50 \times 49}$$

$$P(A) = \frac{2197}{20825}$$ **2**

12. $P(n) : 4^n + 15n - 1$ is divisible by 9

at $n = 1$, $P(1) : 4 + 15 - 1 = 18$ is divisible by 9

So, $P(1)$ is true **1**

Let $P(k)$ is true.

i.e., $4^k + 15k - 1$ is divisible by 9

$\Rightarrow$ $$4^k + 15k - 1 = 9\lambda, \lambda \in N.$$

$\Rightarrow$ $$4^k = 9\lambda - 15k + 1 \quad \text{....(i)}$$ **1**

Now to show $P(k + 1)$ is true.

i.e., $4^{k+1} + 15(k + 1) - 1$ is divisible by 9.

Takeing $$\text{L.H.S.} = 4^{k+1} + 15(k + 1) - 1$$
$$= 4^k.4 + 15k + 15 - 1$$

Using equation (i), we get **1**

$$4(9\lambda - 15k + 1) + 15k + 14$$
$$= 36\lambda - 60k + 4 + 15k + 14$$
$$= 36\lambda - 45k + 18$$
$$= 9(4\lambda - 5k + 2)$$
$$= 9\mu, (\mu = 4\lambda - 5k + 2)$$ **1**

$\Rightarrow P(k + 1)$ is true, when $P(k)$ is true.

So, $P(n)$ is true for all $n \in N$.

OR

$P(n)$ $$\frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \frac{1}{7 \cdot 9} + ... + \frac{1}{(2n+1)(2n+3)}$$
$$= \frac{n}{3(2n+3)}$$

Put $n = 1$,

$$P(1) : \frac{1}{3 \cdot 5} = \frac{1}{3 \cdot 5}$$

So, $P(1)$ is true. **1**

Let $P(k)$ is true.

$$P(k) : \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + ... + \frac{1}{(2k+1)(2k+3)}$$
$$= \frac{k}{3(2k+3)} \quad \text{....(i)}$$

Now prove that $P(k + 1)$ is true.

i.e., $$\frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + ... + \frac{1}{(2k+1)(2k+3)} + \frac{1}{(2k+3)(2k+5)} = \frac{(k+1)}{3(2k+5)}$$

Take L.H.S. $\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+....+\frac{1}{(2k+1)(2k+3)}+\frac{1}{(2k+3)(2k+5)}$ **1**

Using equation (i), we get

$$= \frac{k}{3(2k+3)}+\frac{1}{(2k+3)\,(2k+5)}$$ **1**

$$= \frac{k(2k+5)+3}{3(2k+3)(2k+5)}$$

$$= \frac{2k^2+5k+3}{2(2k+3)(2k+5)}$$

$$= \frac{(2k+3)\,(k+1)}{3(2k+3)(2k+5)}$$

$$= \frac{k+1}{3(2k+5)} = \text{R.H.S.}$$ **1**

Hence, $P(k + 1)$ is true, when $P(k)$ it true.

$\therefore$ $P(n)$ is true for all $n \in N$.

13. $$f(x) = \frac{x^2}{x^2+1},\ x \in R$$ **½**

For domain, as $x^2 \geq 0$

$\Rightarrow$ $x^2 + 1 > 0.$ **½**

$\therefore$ $f(x)$ is defined for all $x \in R.$

$\therefore$ Domain of $f = R$.

For range, let $f(x) = y$

$\Rightarrow$ $$y = \frac{x^2}{x^2+1}$$ **1**

$\Rightarrow$ $$x^2 = \frac{y}{1-y}$$ **½**

$\Rightarrow$ $$x = \sqrt{\frac{y}{1-y}}$$ **1**

$\Rightarrow$ $1 - y > 0 \Rightarrow y < 1$ and $y \geq 0$.

$\therefore$ Range : $0 \leq y < 1$. **½**

14. $f(x) = \sin(5x - 8)$

$\therefore$ $$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin\{5(x+h)-8\}-\sin(5x-8)}{h}$$ **1**

$$= \lim_{h\to 0}\frac{2}{h}\cos\left[\left(\frac{5x+5h-8+5x-8}{2}\right)\cdot\sin\left(\frac{5x+5h-8-5x+8}{2}\right)\right]$$

$$= 2\lim_{h\to 0}\frac{1}{h}\left[\cos\left(5x-8+\frac{5h}{2}\right)\cdot\sin\left(\frac{5h}{2}\right)\right]$$ **1**

$$= 5\lim_{h\to 0}\cos\left((5x-8+\frac{5h}{2}\right)\frac{\sin\left(\frac{5h}{2}\right)}{\left(\frac{5h}{2}\right)}$$ **1**

$$= 5.\cos(5x-8).\ \lim_{h\to 0}\frac{\sin\left(\frac{5h}{2}\right)}{\left(\frac{5h}{2}\right)}$$

$$= 5\cos(5x-8),\ \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$ **1**

15. $$f(x) = \frac{1}{\sqrt{x-[x]}}, x \in R$$

As, $0 \le (x-[x]) < 1,\ \forall\, x \in R$ **1**

But when $x \in Z$

$$x - [x] = 0$$

$\therefore$ $0 < (x-[x]) < 1,\ \forall\, x \in R - Z$ **1**

$\therefore$ Domain of $f = R - Z.$

For range, as $0 < x - [x] < 1,\ \forall\, x \in R - Z$

$\Rightarrow$ $0 < \sqrt{x-[x]} < 1$ **1**

$\Rightarrow$ $1 < \frac{1}{\sqrt{x-[x]}} < \infty$

$\Rightarrow$ $1 < f(x) < \infty$

$\therefore$ Range = $(1, \infty)$. **1**

16. Let two positive integers be x and y.

$\therefore$ According to question,

$$a = \frac{x+y}{2} \quad \text{....(i)}$$

and x, b, c, y are in G.P. **1**

Let r be common ratio.

$\therefore$ $b = xr,\ c = xr^2,\ y = xr^3$

$\Rightarrow$ $r = \left(\frac{y}{x}\right)^{1/3}$

$\Rightarrow$ $b = x\left(\frac{y}{x}\right)^{1/3}$

and $c = x\left(\frac{y}{x}\right)^{2/3}$ **1**

$\Rightarrow$ $b^3 = \frac{x^3y}{x},\ c^3 = \frac{x^3y^2}{x^2}$

$\Rightarrow$ $b^3 + c^3 = \frac{x^3y}{x} + \frac{x^3y^2}{x}$

$= x^2y + xy^2$ **1**

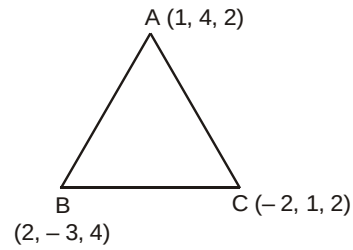
$$b^3 + c^3 = xy\,(x + y)$$

Using (i), $\quad x + y = 2a$

and $\quad xy = bc$

$\therefore \quad \boxed{b^3 + c^3 = 2abc}$ **1**

17. A (1, 4, 2), B (2, – 3, 4), and C (–2, 1, 2) are co-ordinates of vertices of ΔABC, **1**



$$AB^2 = (2 - 1)^2 + (-3 - 4)^2 + (4 - 2)^2$$

$$= 1 + 49 + 4 = 54$$ **1**

$$BC^2 = (-2 - 2)^2 + (1 + 3)^2 + (2 - 4)^2$$

$$BC^2 = 16 + 16 + 4 = 36$$

and $\quad AC^2 = (-2 - 1)^2 + (1 - 4)^2 + (2 - 2)^2$

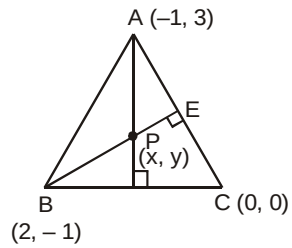
$$= 9 + 9 = 18$$

$\therefore \quad BC^2 + AC^2 = 36 + 18 = 54 = AB^2$ **2**

$\therefore \Delta\, ABC$ is a right triangle.

OR

A (–1, 3), B (2, – 1) and C (0, 0) are the vertices of $\Delta\, ABC$. P is concurrent point of altitude, so it is orthocentre.



Now, $\quad$ slope of $BC = \dfrac{1}{-2}$ **1**

$\Rightarrow \quad$ slope of $AD = 2$

$\therefore$ Equation of AD is $y - 3 = 2\,(x + 1)$

$\Rightarrow \quad 2x - y + 5 = 0$ ....(i) **1**

Now, $\quad$ slope of $AC = \dfrac{-3}{1} = -3$

$\Rightarrow \quad$ slope of $BE = \dfrac{1}{-3}$

$\therefore$ Equation of BE is $\quad y + 1 = \dfrac{1}{3}(x - 2)$

$$3y + 3 = x - 2$$

$\Rightarrow \quad x - 3y = 5$ ....(ii) **1**

On solving equations (i) and (ii), we get

$$x = -4, \text{ and } y = -3.$$

Co-ordinates of orthocentre = P (– 4, – 3). **1**

18.

Class	f_i	x_i	$u_i = \dfrac{x_i - 105}{30}$	u_i^2	$f_i\, u_i$	$f_i\, u_i^2$
0 – 30	2	15	–3	9	–6	18
30 – 60	3	45	–2	4	–6	12
60 – 90	5	75	–1	1	–5	5
90 – 120	10	105 = a	0	0	0	0
120 – 150	3	135	1	1	3	3
150 – 180	5	165	2	4	10	20
180 – 210	2	195	3	9	6	18
Total	**30**				**2**	**76**

$\therefore$ Variance $= \sigma^2 = \dfrac{h^2}{N^2}\ [N.\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2]$

Here $h = 30,\ N = 30.$ **2**

$\therefore$ $\sigma^2 = \dfrac{30^2}{30^2}\left[30 \times 76 - (2)^2\right]$

$= [2280 - 4]$

$\boxed{\sigma^2 = 2276}$ **2**

19. We have word DAUGHTER which consists 3 vowels and 5 consonants.

$\therefore$ Number of words consisting 2 vowels and 3 consonants

$= {}^3C_2 \times {}^5C_3 \times 5\,!$ **2**

$= \dfrac{3 \times 2}{2 \times 1} \times \dfrac{5 \times 4 \times 3}{3 \times 2 \times 1} \times 120$

$= 3600.$ **2**

Section - C

20. L.H.S. $= \dfrac{\cos 4x + \cos 2x + \cos 3x}{\sin 4x + \sin 2x + \sin 3x}$

$= \dfrac{2\cos\left(\dfrac{4x+2x}{2}\right)\cdot\cos\left(\dfrac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\dfrac{4x+2x}{2}\right)\cdot\cos\left(\dfrac{4x-2x}{2}\right) + \sin 3x}$ **2**

$= \dfrac{2\cos 3x.\cos x + \cos 3x}{2\sin 3x.\cos x + \sin 3x}$ **2**

$= \dfrac{\cos 3x\,(2\cos x + 1)}{\sin 3x\,(2\cos x + 1)}$ **2**

$= \cos 3x =$ R.H.S. **Hence proved.**

21. A = Number of people who read English news-paper **1**

B = Number of people who read Hindi news-paper **1**

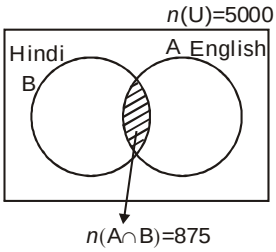
$\therefore$ According to question, **1**

$n(A) = 2250,\ n(B) = 1750$ and $n(A \cap B) = 875.$

$\therefore$ $n\ (A \cup B) = n(A) + n(B) - n\ (A \cap B)$

$= 2250 + 1750 - 875$

$= 400 - 875 = 3125$ **1**



$\therefore$ Number of people who neither read Hindi nor English

$= n(U) - n\ (A \cup B)$

$= 5000 - 3125 = 1875.$ **1**

Number of people who reads only English news-paper

$= 2250 - 875$

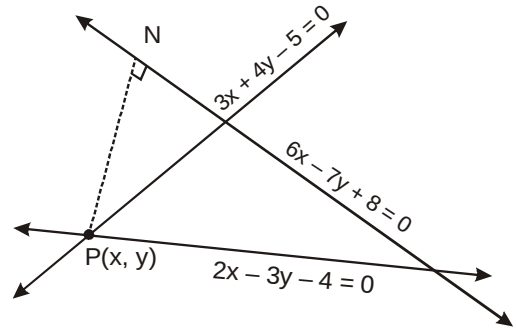
$= 1375.$ **1**

22. Equations of two straight roads are

$2x - 3y - 4 = 0$ ...(i)

and $3x + 4y - 5 = 0$ ...(ii) **1**

Family is standing at the point *P*.



$\therefore$ On solving equations (i) and (ii), we get

$$x = \frac{31}{71},\ y = \frac{-2}{17}$$

$\therefore \quad P\left(\frac{31}{17}, \frac{-2}{17}\right)$ **1**

Because family is at $P\left(\frac{31}{17}, \frac{-2}{17}\right)$ want to reach the path

$$6x - 7y + 8 = 0 \qquad ...(iii)$$ **½**

in least time.

So they have to walk along *PN*.

Now, slope of path (iii) $= \frac{6}{7}$ **1**

$\therefore$ Slope of $PN = \frac{-7}{6}$

$\therefore$ Equation of $PN = y + \frac{2}{17} = \frac{-7}{6}\left(x - \frac{31}{17}\right)$

$\Rightarrow \quad \frac{17y+2}{17} = \frac{-7}{6}\left(\frac{17x-31}{17}\right)$ **1**

$\Rightarrow \quad 102y + 12 = -119x + 217$

$\Rightarrow \quad 119x + 102y = 217 - 12$ **½**

$\Rightarrow \quad 119x + 102y = 205.$

So, he should follow the path given by $119x + 102y = 205$. **1**

Because of unavailability of road transport, they have less time to reach to station, that is why they have decided to took the shortest path.

23. The expansion is $(1 + x)^n$

$\therefore \quad T_{r+1} = {}^nC_r\, x^r$

$\therefore$ Coefficients of $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms are ${}^nC_{r-1}$, ${}^nC_r$ and ${}^nC_{r+1}$ **1**

According to question,

$${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 45 : 120 : 210$$

$\Rightarrow \quad \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{45}{120}$ **1**

$\Rightarrow \quad \frac{(n)!}{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{45}{120}$

$\Rightarrow \quad 3x - 11r + 3 = 0 \qquad ....(i)$ **1**

Also, $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{120}{210}$

$\Rightarrow \quad \frac{n!}{r!(n-r)!} \times \frac{(r+1)!(n-r-1)!}{n!} = \frac{4}{7}$ **1**

$\Rightarrow \quad 4n - 11r - 7 = 0 \qquad ....(ii)$ **1**

On solving (i) and (ii), we get

$$n = 10 \text{ and } r = 3$$

$\therefore \quad \boxed{n = 10}$ **1**

OR

$(52)^4 = (50 + 2)^4$ **1**

Using Binomial theorem, we get

$= (50)^4 + {}^4C_1\, 50^3 \times 2 + {}^4C_2 \times 50^2 \times 2^2 + {}^4C_3 \times 50 \times 2^3 + {}^4C_4 \times 2^4$ **2**

$= 6250000 + 8 \times 125000 + 24 \times 2500 + 32 \times 50 + 16$ **1**

$= 6250000 + 1000000 + 60000 + 1600 + 16$ **1**

$= 73,11,616$ **2**

24. Given inequations are

$3x + 2y \leq 60$ ....(i)

$x + 3y \leq 30,\ x \geq 0,\ y \geq 0$ ....(ii)

Take $3x + 3y = 60$

$\therefore$ At $x = 0,\ y = 20$ **1**

and $y = 0,\ x = 20$

At (0, 0); 0 + 0 < 60, (True)

i.e., (0, 0) is included. **1**

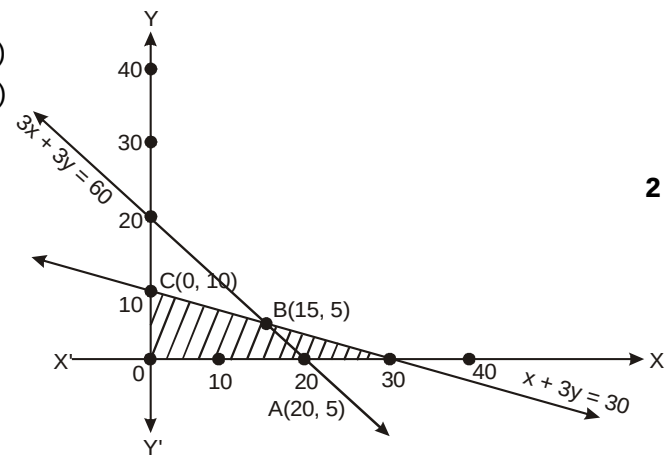
Now take $x + 3y = 30$

$\therefore$ At $x = 0,\ y = 10$

and $y = 0,\ x = 30.$

At (0, 0); 0 + 0 < 30, (True)

So, origin is also included. **2**



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So, shaded region *OABC* is the solution region.

25.

$$\text{L.H.S.} = \frac{1\cdot 2^2 + 2\cdot 3^2 + ... + n(n+1)^2}{1^2\cdot 2 + 2^2\cdot 3 + ... + n^2(n+1)}$$

$$= \frac{\Sigma[n(n+1)^2]}{\Sigma[n^2(n+1)]}$$ **1**

$$= \frac{\Sigma[n^3 + 2n^2 + n]}{\Sigma[n^3 + n^2]}$$

$$= \frac{\Sigma n^3 + 2\Sigma n^2 + \Sigma n}{\Sigma n^3 + \Sigma n^2}$$ **1**

$$= \frac{\left[\frac{n^2(n+1)^2}{4} + 2 \times \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]}{\left[\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}\right]}$$

$$= \frac{\left(\frac{3n^2(n+1)^2 + 4n(n+1)(2n+1) + 6n(n+1)}{12}\right)}{\left(\frac{3n^2(n+1)^2 + 2n(n+1)(2n+1)}{12}\right)}$$ **1**

$$= \frac{n(n+1)(3n^2 + 3n + 8n + 4 + 6)}{n(n+1)(3n^2 + 3n + 4n + 2)}$$ **1**

$$= \frac{3n^2 + 11n + 10}{3n^2 + 7n + 6}$$ **1**

$$= \frac{(3n+5)(n+2)}{(3n+1)(n+2)}$$

$$= \frac{3n+5}{3n+1} = \text{R.H.S.}$$ **Hence proved.** **1**

26. Given that,

$(a + ib)(c + id)(e + if)(g + ih) = A + iB$ **1**

$\therefore \quad (a - ib)(c - id)(e - if)(g - (ih) = A - iB$ **2**

$\therefore \quad (A + iB)(A - iB) = (a + ib)(a - ib)(c + id)$

$(c - id)(e + if)(e - if)(g + ih)(g - ih)$ **2**

$A^2 + B^2 = (a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2).$ **1**

OR

We have $\left|\frac{z+2-i}{z+5+4i}\right| = 5$, where z is only complex number.

$\Rightarrow \quad \left|\frac{z+2-i}{z+5+4i}\right|^2 = 25$ **1**

$\left|\frac{x+iy+2-i}{x+iy+5+4i}\right|^2 = 25$ **1**

$\left|\frac{(x+2)+i(y-1)}{(x-5)+i(y+4)}\right|^2 = 25$

$\Rightarrow$ As, $|z|^2 = z\bar{z}$ **1**

$\therefore \quad \left(\frac{(x+2)+i(y+1)}{(x+5)+i(y+4)}\right)\left(\frac{(x+2)+i(y-1)}{(x+5)+i(y+4)}\right) = 25$

$\Rightarrow \quad \frac{[(x+2)+i(y-1)][(x+2)-i(y+1)]}{[(x+5)+i(y+4)][(x+5)-i(y+4)]} = 25$ **1**

$\Rightarrow \therefore \quad (x + 2)^2 + (y - 1)^2 = 25\,[(x + 5)^2 + (y + 4)^2]$

$\Rightarrow \quad x^2 + 4 + 4x + y^2 + 1 - 2y = 25\,(x^2 + 25 + 10x + y^2 + 16 + 8y]$

$\Rightarrow \quad 25x^2 + 25y^2 + 250x + 200y + 1025 - x^2 - y^2 - 4x + 2y - 5 = 0$ **1**

$\Rightarrow \quad 24x^2 + 24y^2 + 246x + 202y + 1020 = 0$

$\Rightarrow \quad 12x^2 + 12y^2 + 123x + 101y + 510 = 0.$ **1**

❑❑

SAMPLE QUESTION PAPER–7

Section-A

1. We have $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 5, 7\}$

$\therefore$ R = A relation "is less than" from A to B **1**

$R = \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 5), (3, 7), (4, 5), (4, 7)\}$.

2. $5x - 3 < 3x + 1$

$\Rightarrow$ $2x < 4$

$\Rightarrow$ $x < 2$.

(i) When x is an integer, then $x = \{.... -3, -2, -1, 0, 1\}$ **½**

(ii) When x is real number, then $-\infty < x < 2$. **½**

3. Equation of line is $y - 3 = \frac{1}{2}(x+4)$

$\Rightarrow$ $x + 4 - 2y + 6 = 0$

$\Rightarrow$ $x - 2y + 10 = 0$. **1**

4. Centre of circle = $(-2, 3)$, $r = 4$

$\therefore$ Equation of circle is $(x + 2)^2 + (y - 3)^2 = 4^2$

$\Rightarrow$ $x^2 + y^2 + 4x - 6y - 6 = 0$. **1**

5.
$$\lim_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x} = \lim_{x\to 0}\left[\frac{2\cos\left(\frac{2+x+2-x}{2}\right)\cdot\sin\left(\frac{2+x-2+x}{2}\right)}{x}\right]$$
$$= \lim_{x\to 0} 2\cos 2\cdot\frac{\sin x}{x}$$
$$= 2\cos 2\cdot\lim_{x\to 0}\left(\frac{\sin x}{x}\right)$$
$$= 2\cos 2.$$ **1**

6. It is false that, the sum of 2 and 3 is 6. **1**

Section-B

7.
$$\text{L.H.S.} = \cos 2A\cdot\cos\frac{A}{2} - \cos 3A\cdot\cos\frac{9A}{2}$$
$$\frac{1}{2}\left[2\cos 2A\cdot\cos\frac{A}{2} - 2\cos 3A\cdot\cos\frac{9A}{2}\right]$$ **1**
$$= \frac{1}{2}\left[\cos\left(2A+\frac{A}{2}\right)+\cos\left(2A-\frac{A}{2}\right) - \cos\left(3A+\frac{9A}{2}\right)-\cos\left(3A-\frac{9A}{2}\right)\right]$$ **1**
$$= \frac{1}{2}\left(\cos\frac{5A}{2}+\cos\frac{3A}{2}-\cos\left(\frac{15A}{2}\right)-\cos\left(\frac{-3A}{2}\right)\right)$$

But $\cos(-A) = \cos A$
$$= \frac{1}{2}\left(\cos\frac{5A}{2}-\cos\frac{15A}{2}\right)$$ **1**

$$= \frac{1}{2} \times 2\sin\frac{1}{2}\left(\frac{5A}{2} + \frac{15A}{2}\right) \cdot \sin\left(\frac{15A}{2} - \frac{5A}{2}\right)$$

$$= \sin(4A).\sin\left(\frac{5A}{2}\right) = \text{R.H.S.}$$ **1**

Hence proved

8. A = Number of students who takes tea

B = Number of students who takes coffee **1**

According to question, $n(A) = 150$, $n(B) = 225$ **1**

$n(A \cap B) = 100$

$\therefore$ $n(A \cup B) = 150 + 225 - 100$ **1**

$= 375 - 100$

$n(A \cup B) = 275.$

$\therefore$ Number of students who neither take tea nor coffee

$= n(U) - n(A \cup B)$

$= 600 - 275$

$= 325.$ **1**

OR

$$f(x) = \frac{(x-4)}{|x-4|}$$

For Domain $|x - 4| \neq 0$ *i.e.,* $x \neq 4$

$\therefore$ Domain $= R - \{4\}$ **2**

Range : When $x \geq 4$; $f(x) = \dfrac{(x-4)}{(x-4)} = 1$

and when $x < 4$, $f(x) = \dfrac{(x-4)}{-(x-4)} = -1$

$\therefore$ Range of $f = \{-1, 1\}$. **2**

9. It is given that $a \tan\theta + b \sec\theta = c$

$\Rightarrow$ $(a \tan\theta - c) = -b \sec\theta$ **1**

$\Rightarrow$ $a^2 \tan^2\theta + c^2 - 2ac \tan\theta = b^2 \sec^2\theta$

$\Rightarrow$ $b^2(1 + \tan^2\theta) - a^2 \tan^2\theta - c^2 + 2ac \tan\theta = 0$

$\Rightarrow$ $(b^2 - a^2)\tan^2\theta + 2ac \tan\theta + b^2 - c^2 = 0.$ **1**

As α and β are its solutions

$\therefore$ $\tan\alpha$ and $\tan\beta$ are their roots.

$\therefore$ $\tan\alpha + \tan\beta = \dfrac{-2ac}{b^2 - a^2}$

and $\tan\alpha \tan\beta = \dfrac{b^2 - c^2}{b^2 - a^2}$ **1**

$\therefore$ $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$$= \frac{\left(\dfrac{-2ac}{b^2 - a^2}\right)}{1 - \left(\dfrac{a^2 - c^2}{b^2 - a^2}\right)}$$

$$= \left[\frac{\frac{-2ac}{b^2-a^2}}{\frac{(c^2-a^2)}{b^2-a^2}}\right]$$

$$\boxed{\tan(\alpha+\beta) = \frac{2ac}{a^2-c^2}}$$ **1**

10. As $$p + iq = \frac{(a+i)^2}{2a-i}$$

$\Rightarrow$ $$p - iq = \frac{(a-i)^2}{2a+i}$$ **1**

$\therefore$ $$(p + iq)(p - iq) = \frac{(a+i)^2}{2a-i} \times \frac{(a-i)^2}{(2a+i)}$$ **2**

$$p^2 + q^2 = \frac{(a^2+1)^2}{4a^2+1},$$ **Hence proved.** **1**

OR

$$z = 1 + i\sqrt{3}$$ (given)

Put $$1 = r\cos\theta$$ ....(i)

and $$\sqrt{3} = r\sin\theta$$ ....(ii)

Square and adding, we get

$$r^2 = 4 \Rightarrow r = 2$$ **1**

and $$\tan\theta = \sqrt{3} = \tan\frac{\pi}{3}$$

$\Rightarrow$ $$\theta = \arg(z) = \frac{\pi}{3}$$ **2**

$\therefore$ $$1 + i\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right).$$ **1**

11. We have

$$\tan(\alpha+\beta) = n\tan(\alpha-\beta)$$

$\Rightarrow$ $$\frac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)} = \frac{n}{1}$$

Using C and D, we get

$$\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{\tan(\alpha+\beta)-\tan(\alpha-\beta)} = \frac{n+1}{n-1}$$ **2**

$\Rightarrow$ $$\frac{\left(\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}+\frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}\right)}{\left(\frac{\sin(\alpha+\beta)}{\cos(\alpha-\beta)}-\frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}\right)} = \frac{n+1}{n-1}$$ **1**

$\Rightarrow$ $$\frac{\sin(\alpha+\beta).\cos(\alpha-\beta)+\cos(\alpha+\beta).\sin(\alpha-\beta)}{\sin(\alpha+\beta).\cos(\alpha-\beta)+\cos((\alpha+\beta).\sin(\alpha-\beta)} = \frac{n+1}{n-1}$$ **1**

$$\Rightarrow \quad \frac{\sin(\alpha+\beta+\alpha-\beta)}{\sin(\alpha+\beta-\alpha+\beta)} = \frac{n+1}{n-1}$$

$$\Rightarrow \quad \frac{\sin(2\alpha)}{\sin(2\beta)} = \frac{n+1}{n-1}$$

$$\Rightarrow \quad (n+1)\sin 2\beta = (n-1)\sin 2\alpha.$$ **Hence proved.**

12. We have 6 red, 5 white and 5 blue balls, then 9 balls are to be select with 3 balls of each colour.

$\therefore$ Total ways = ${}^6C_3 \times {}^5C_3 \times {}^5C_3$ **2**

$$= \frac{6\times5\times4}{3\times2\times1}\times\frac{5\times4\times3}{3\times2\times1}\times\frac{5\times4\times3}{3\times2\times1}$$

$= 20 \times 10 \times 10$

$= 2000.$ **2**

OR

Let number of persons present in marriage anniversary party = n

Everybody shakes hand to each other.

$\therefore$ Total number of shake = ${}^nC_2$ **1**

$\therefore$ According to question,

$${}^nC_2 = 66$$

$$\Rightarrow \quad n(n-1) = 132$$

$$\Rightarrow \quad n^2 - n - 132 = 0$$

$$\Rightarrow \quad (n-12)(n+11) = 0$$ **1**

$$\Rightarrow \quad n = 12,\ n = -11 \text{ (not possible)}$$

$\therefore$ Total persons present in the room = 12. **2**

Shaking hand is the begining of friendship, so it is healthy thinking to establish good relationship.

13. As $${}^nP_5 = 42\cdot{}^nP_3$$

$$\Rightarrow \quad \frac{n!}{(n-5)!} = 42\cdot\frac{n!}{(n-3)}$$ **½**

$$\Rightarrow \quad \frac{1}{(n-5)!} = \frac{42}{(n-3)(n-4)(n-5)!}$$ **1½**

$$\Rightarrow \quad (n-3)(n-4) = 4$$ **½**

$$\Rightarrow \quad n^2 - 7n + 12 - 42 = 0$$

$$\Rightarrow \quad n^2 - 7n - 30 = 0$$ **½**

$$\Rightarrow \quad (n-10)(n+3) = 0$$

$$\Rightarrow \quad n = 10,\ n = -3 \text{ (not possible)}$$ **1**

14. Two unbiased dices are thrown.

$\therefore$ S = {(1, 1), (1, 2), (1, 3), ..., (6, 4), (6, 5), (6, 6)}

i.e., Total sample points = 36 **½**

A = Event of getting a doublet or a total 10

$\therefore$ A ={(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (4, 6), (6, 4)} **1**

$\therefore$ $$P(A) = \frac{8}{36} = \frac{2}{9}$$ **½**

$\therefore$ P (Neighter a doublet nor a total 10 will appear)

$$= 1 - \frac{2}{9}$$ **½**

i.e., $$\boxed{P(\bar{A}) = \frac{7}{9}}$$ **½**

15. We have 2 men and 2 woman, A = committee of 2 is selected having one man *i.e.,* P (selecting 1 man and 1 woman) **1**

$$P(A) = \frac{{}^2C_1 \times {}^2C_1}{{}^4C_2} \quad \textbf{2}$$

$$= \frac{2 \times 2 \times 2}{4 \times 3} = \frac{2}{3}. \quad \textbf{1}$$

16.

Class	**Frequency (f)**	x	$u_i = \frac{x_i - 65}{10}$	u_i^2	$f_i u_i$	$f_i u_i^2$
30 – 40	3	35	–3	9	–9	27
40 – 50	7	45	–2	4	–14	28
50 – 60	12	55	–1	1	–12	12
60 – 70	15	65 = a	0	0	0	0
70 – 80	8	75	1	1	8	8
80 – 90	3	85	2	4	6	12
90 – 100	2	95	3	9	6	18
Total	**50**				**–15**	**105**

Here, $a = 65,\ h = 10,\ \Sigma f = 50$

$\therefore$ $$\text{Mean} = 65 + \frac{-15}{50} \times 10 \quad \textbf{1}$$

$$= 65 - 3 = 62$$

and $$\text{Variance} = h^2\left[\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2\right]$$

$$= 100\left[\frac{105}{50} - \left(\frac{-15}{50}\right)^2\right] \quad \textbf{1}$$

$$= 100\left[\frac{5250 - 225}{50 \times 50}\right]$$

$$= \frac{5025}{25} = 201 \quad \textbf{2}$$

$$\boxed{\text{Variance} = 201, \text{ and Mean} = 62}$$

17. Equation of hyperbola is $16x^2 - 9y^2 = 144$

$\Rightarrow$ $$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

$\therefore$ $a^2 = 9 \Rightarrow a = 3$ and $b = 4$ **1**

Length of transversal = $2a = 6$

Length of conjugate axis = $2b = 8$

$$c^2 = a^2 + b^2 = 9 + 16 = 25$$

$\Rightarrow$ $c = 5$ **1**

$\therefore$ $$e = \frac{5}{3}$$

foci = $(\pm c, 0) = (\pm 5, 0)$ **1**

and vertices = $(\pm a, 0), = (\pm 3, 0)$ **1**

18. $$f(x) = \frac{2x+3}{x-2}$$

Using first principle,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{2(x+h)+3}{x+h-2} - \frac{(2x+3)}{x-2}\right] \quad \mathbf{1}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{(2x+2h+3)(x-2)-(x+h-2)(2x+3)}{(x+h-2)(x-2)}\right] \quad \mathbf{1}$$

$+ h\ (2x - 4 - 2x - 3$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{(2x+3)(x-2)-(2x+3)(x-2)}{(x+h-2)(x-2)}\right]$$

$$= \lim_{h\to 0} \left[\frac{h(2x-4-2x-3)}{h(x+h-2)(x-2)}\right] \quad \mathbf{1}$$

$$= \frac{-7}{(x-2)^2}$$

$\therefore$ $$\boxed{f'(x) = \frac{-7}{(x-2)^2}} \quad \mathbf{1}$$

19. As roots of $x^2 - 3x + p = 0$ are a and b.

$\therefore$ $a + b = 3$ and $ab = p$ ....(i) **½**

and $x^2 - 12x + q = 0$ have c and d roots.

So, $c + d = 12$, and $cd = q$ ....(ii) **1**

But a, b, c and d are in G.P.

$\therefore$ $$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = r \text{ (common ratio)}$$

$\Rightarrow$ $b = ar$, $c = ar^2$ and $d = ar^3$ ....(iii) **1**

From (i), (ii) and (iii), we get

$$a \cdot ar = p \Rightarrow a^2 r = p$$

and $$ar^2 \cdot ar^3 = q = a^2 r^5 = q \quad \mathbf{½}$$

$\Rightarrow$ $$\frac{a^2 r^5}{a^2 r} = \frac{q}{p} \Rightarrow r^4 = \frac{q}{p}$$ ....(iv) **1**

Again, $a + ar = 3 \Rightarrow a(1 + r) = 3$

and $ar^2 + ar^3 = 12 \Rightarrow ar^2 (1 + r) = 12$

On dividing these, we get

$$r^2 = 4 \Rightarrow \frac{q}{p} = (4)^2 = 16 \quad \text{(from iv)}$$

Applying componendo and dividendo, we get

$$\frac{q+p}{q-p} = \frac{16+1}{16-1} = \frac{17}{15} \quad \mathbf{½}$$

Teacher can assess the ability of learning and understanding on the basis of assesment he can guide and encourage them. **½**

Section – C

20. Take, $6^n = (1 + 5)^n$ **1**

$6^n = 1 + {}^nC_1 (5) + {}^nC_2 + {}^nC_3\, 5^3 + ...$

$\Rightarrow$ $6^n = 1 + 5n + 5^2\, [{}^nC_2 . (5) + {}^nC_3\, 5 + ...]$

$\Rightarrow$ $6^n - 5n - 1 = 25\, [{}^nC_2 + 5.\, {}^nC_3 + ...]$ **1**

$= 25\lambda\ ;\ \lambda = [{}^nC_2 + 5\, {}^nC_3 + ...]$ **1**

$\Rightarrow$ $6^n - 5n - 1 = 25\lambda$ **2**

$\therefore$ $6^n - 5n - 1$ is divisible by 25. **1**

21. Expansion $= \left(2^{1/4} + \dfrac{1}{3^{1/4}}\right)^n$ **1**

$= \left[2^{1/4} + 3^{-1/4}\right]^n$

$\therefore$ 5th term from beginning $= {}^nC_4 \left(2^{\frac{1}{4}}\right)^{n-4} \times \left(3^{-\frac{1}{4}}\right)^4$

$= {}^nC_4\, 2^{\left(\frac{n-4}{4}\right)} \times 3^{-1}$ ....(i) **1**

5th term from end $= {}^nC_4 \left(3^{-\frac{1}{4}}\right)^{x-4} \times \left(2^{\frac{1}{4}}\right)^4$ **1**

and $= {}^nC_4\, 3^{-\frac{(x-4)}{4}} (2)^1$ ....(ii)

According to question,

$$\frac{{}^nC_4\, 2^{\frac{n-4}{4}} \times 3^{-1}}{{}^nC_4\, 3^{-\frac{(n-4)}{4}} \times 2} = \frac{(6)^{1/2}}{1}$$ **1**

$\Rightarrow$ $2^{\left(\frac{n-4}{4} - 1\right)} \times 3^{\frac{n-4}{4} - 1} = (6)^{\frac{1}{2}}$

$\Rightarrow$ $(6)^{\left(\frac{n-8}{4}\right)} = 6^{\frac{1}{2}}$ **1**

$\Rightarrow$ $\dfrac{n-8}{4} = \dfrac{1}{2}$ **1**

$\Rightarrow$ $\boxed{n = 10}$

OR

Expansion $= (1 + x)^n$

$\therefore$ 3 consecutive coeficients are

${}^nC_{r-1}$, ${}^nC_r$ and ${}^nC_{r+1}$ **1**

According to question

${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 6 : 33 : 110$

$$\frac{n!}{(r-1)!\,(n-r+1)} \times \frac{r!(n-r)!}{n!} = \frac{6}{33}$$ **1**

$\Rightarrow$ $\dfrac{r}{n-r+1} = \dfrac{2}{11}$

$$11r = 2n - 2r + 2$$

$\Rightarrow$ $$2n - 13r + 2 = 0 \quad \text{....(i)}$$ **1**

and $$\frac{n!}{r!(n-r)} \times \frac{(r+1)!(n-r-1)!}{n!} = \frac{33}{110}$$

$\Rightarrow$ $$\frac{(r+1)}{(n-r)} = \frac{3}{10}$$

$\Rightarrow$ $$3n - 3r = 10r + 10$$ **1**

$\Rightarrow$ $$3n - 13r - 10 = 0 \quad \text{....(ii)}$$ **1**

On solving equations (i) and (ii), we get

$$n = 12 \text{ and } r = 2.$$ **1**

22. $P(x) : 10^{2n-1} + 1$ is divisible by 11.

Put $x = 1$, $P(1) = 10^{2-1} + 1 = 11$ is divisible by 11

i.e., $P(1)$ is true. **1**

Let $P(k)$ is true

$$P(k) : 102^{k-1} + 1 = 11\lambda; (\lambda \in N)$$

$\Rightarrow$ $$10^{2k} = 10\,(11\,\lambda - 1) \quad \text{....(i)}$$ **1**

Now to prove $P(k+1)$ is true.

i.e. $10^{2(k+1)-1} + 1$ is divisible by 11

Take $$\text{L.H.S.} = 10^{2k+2-1} + 1$$ **1**

$$= 10^{2k+1} + 1$$

Using equation (i), we get

$$10 \times 10\,(11\lambda - 1) + 1$$ **1**

$\Rightarrow$ $$1100\,\lambda - 100 + 1 = 1100\,\lambda - 99$$

$\Rightarrow$ $$11\,(100\,\lambda - 9) = 11\mu, \quad (\because \mu = 100\,\lambda - 9)$$

i.e., $P(k + 1)$ is true. **2**

Hence, $P(n)$ is true for all $n \in N$.

23. A = number of players in Basketball team

B = number of players in Hockey team

C = number of players in Football team **1**

$$n(A) = 21,\ n(B) = 26,\ n(C) = 20$$

$$n(B \cap A) = 14,\ n(B \cap C) = 15$$ **1**

$$n(A \cap C) = 12, \text{ and } n(A \cap B \cap C) = 8$$ **1**

$\therefore$ $$n(A \cup B \cup C) = n(A) + n(B) + n(C)$$

$$- n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$ **1**

$$= 21 + 26 + 20 - 14 - 15 - 12 + 8$$

$$= 67 + 8 - 41$$

$$= 75 - 41$$

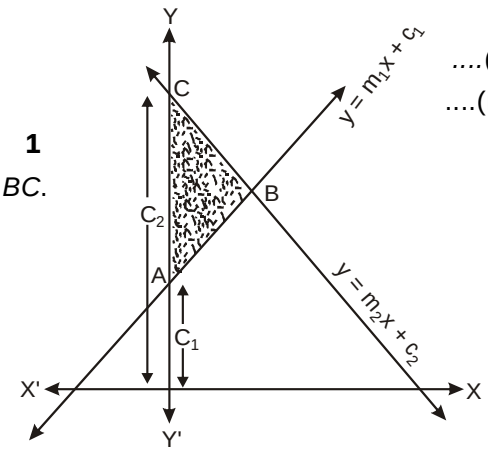
$$n(A \cup B \cup C) = 34$$ **2**

So, number of all players is 34.

24. We have lines

$$y = m_1x + c_1, \quad ....(i)$$
$$y = m_2x + c_2 \quad ....(ii)$$

and $x = 0.$ **1**



On drawing these equations graphically we get a $\Delta\, ABC$.

On solving equations, (i) and (ii), we get

$$m_1x + c_1 = m_2x + c_2 \quad \textbf{1}$$

$$\Rightarrow \quad x = \frac{c_2 - c_1}{m_1 - m_2}$$

$$\therefore \quad y = \frac{m_1(c_2 - c_1)}{m_1 - m_2} + c_1$$

$$y = \frac{m_1c_2 - m_1c_1 + m_1c_1 - m_2c_1}{(m_1 - m_2)}$$

$$y = \frac{(m_1c_2 - m_2c_1)}{m_1 - m_2} \quad \textbf{1}$$

$\therefore$ Co-ordinates of A $(0, c_1)$,

$$B\left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2}\right), C(0, c_2) \quad \textbf{1}$$

$$\therefore \quad \text{Ar } (ABC) = \frac{1}{2}\left|0\left(\frac{m_1c_2 - m_2c_1}{m_1 - m_2} - c_2\right) + \frac{(c_2 - c_1)}{(m_1 - m_2)}(c_2 - c_1) + 0\left(c_1 - \frac{m_1c_2 - m_2c_1}{m_1 - m_2}\right)\right| \quad \textbf{1}$$

$$= \frac{1}{2}\left|\frac{(c_2 - c_1)^2}{m_1 - m_2}\right|$$

$$\therefore \quad \boxed{\text{Ar } (ABC) = \frac{(c_2 - c_1)^2}{2|\, m_1 - m_2 \,|}}$$

Hence proved. **1**

OR

Equation of line is, $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ ....(i)

$p_1 = \perp$ distance from $\left(\sqrt{a^2 - b^2}, 0\right)$ on (i) **1**

$$= \frac{\left|\frac{\sqrt{a^2 - b^2}}{a}\cos\theta + 0 - 1\right|}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}} \quad ....(ii) \quad \textbf{1}$$

and $p_2 = \perp$ distance of line (i) from $\left(-\sqrt{a^2 - b^2}, 0\right)$

$$= \frac{\left|\frac{-\sqrt{a^2 - b^2}}{a}\cos\theta + 0 - 1\right|}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}} \quad ....(iii) \quad \textbf{1}$$

$\therefore$ Product of $\perp$ distance using equations (ii) and (iii)

$$p_1p_2 = \frac{-\left(\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right)\left(\frac{+\sqrt{a^2-b^2}}{a}\cos\theta + 1\right)}{\left(\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}\right)}$$ **1**

$$p_1p_2 = \frac{-\left(\frac{a^2-b^2}{a^2}\cos^2\theta - 1\right)}{\left(\frac{(b^2\cos^2\theta + a^2\sin^2\theta)}{a^2b^2}\right)}$$ **1**

$$= \frac{(a^2 - a^2\cos^2\theta + b^2\cos^2\theta)a^2b^2}{(b^2\cos^2\theta + a^2\sin^2\theta)a^2}$$

$$p_1p_2 = \frac{(a^2\sin^2\theta + b^2\cos^2\theta)b^2}{(b^2\cos^2\theta + a^2\sin^2\theta)} = b^2.$$

$\therefore$ Product of perpendicular distances is b^2. **1**

25. Let a be first term and r be common ratio of G.P.

So first 4 terms are a, ar, ar^2, ar^3. **1**

According to question,

$$a_3 = a + 9$$
$$ar^2 = a + 9$$
$$\Rightarrow \quad a(r^2 - 1) = 9 \quad \text{....(i)}$$ **1**

and
$$a_2 = 94 + 18$$
$$ar = ar^3 + 18$$
$$\Rightarrow \quad -ar(r^2 - 1) = 18 \quad \text{....(ii)}$$ **1**

From (i) and (ii), we get

$$\frac{-ar(r^2-1)}{a(r^2-1)} = \frac{18}{9}$$

$$\Rightarrow \quad r = -2$$ **1**

$\therefore$ From (i), $a = \frac{9}{4-1} = 3$ **1**

$\therefore$ Four terms of G.P. = a = 3

$$ar = 3 \times (-2) = -6$$
$$ar^2 = 3 \times (-2)^2 = 12$$

and $ar^3 = 3 \times (-2)^3 = -24$

$\therefore$ Four terms are 3, – 6, 12, – 24. **1**

26. We have linear inequations

$$x - 2y \le 3 \quad \text{....(i)}$$
$$3x + 4y \ge 12 \quad \text{....(ii)}$$

and $x \ge 1$ and $y \ge 1$.

Take $x - 2y = 3$ **1**

At $x = 0$, $y = -3/2$

and at $y = 0$, $x = 3$

At (0, 0), $0 - 0 < 3$ (True) **1**

So origin is included.

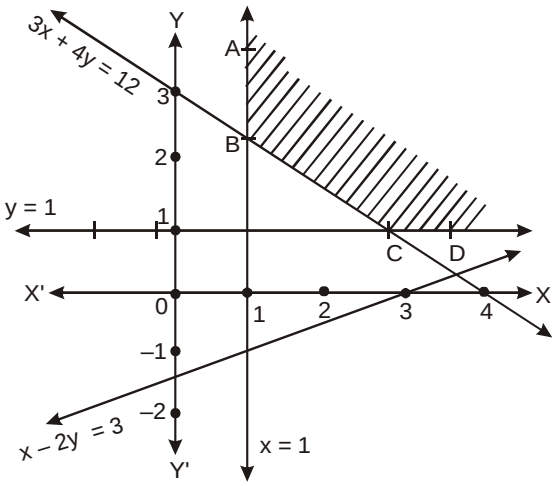
Now take $3x + 4y = 12$ **1**

At $x = 0$, $y = 3$

and at $y = 0$, $x = 4$

At (0, 0); $0 + 0 > 12$ (False)

So origin is not included.



2

$x = 1$ is a line parallel to y-axis at (0, 0) is not included and $y = 1$ is a line parallel to x-axis the shaded region is the solution region. **1**

❑❑

SAMPLE QUESTION PAPER–8

Section-A

1. Given that $A = \{x : x^2 < 70, x \in N\}$

$\therefore$ Roster form $= \{1, 2, 3, 4, 5, 6\ 7, 8\}$ **1**

2. $R = \{(x, y) \in A \times B, x \in A, y \in B, x > y\}$

$\therefore$ $R = \{(3, 2), (5, 2), (5, 4)\}$ **1**

3. $\frac{d}{dx}\sin^2 x = 2 \sin x \cos x = \sin 2x$ **1**

4. Foci of ellipse ($\pm$ 5, 0) and $e = \frac{1}{2}$

i.e., $c = 5 \Rightarrow e = \frac{c}{a}$

$\Rightarrow$ $\frac{1}{2} = \frac{5}{a} \Rightarrow a = 10 \Rightarrow a^2 = 100$

and $b^2 = 10^2 - 5^2 = 75$

$\therefore$ Equation of ellipse is $\frac{x^2}{100} + \frac{y^2}{75} = 1.$ **1**

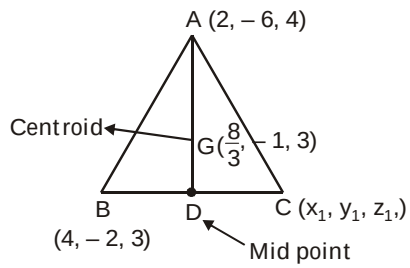
5. *AD* is median and *G* is centroid

$\therefore$ $\frac{2+4+x_1}{3} = \frac{8}{3}$

$\Rightarrow$ $x_1 = 2$

Similary $y_1 = 5$ and $z_1 = 2$.

$\therefore$ Co-ordinates of centroid = (2, 5, 2). **1**



6. It is not the case that 2 is greater than 7. **1**

Section - B

7. L.H.S. = cos 10° cos 30° cos 50° cos 70°

$= \frac{1}{2}$ [2 cos 10° cos 50°] cos 70° cos 30° **1**

$= \frac{1}{2} \times \frac{\sqrt{3}}{2}$ [(cos 60° + cos 40°) cos 70°],

$\because$ 2 cos *A* cos *B* = cos (*A* + *B*) + cos (*A* – *B*)

$= \frac{\sqrt{3}}{2 \times 4}\left[2 \times \frac{1}{2}\cos 70° + 2\cos 40° \cos 70°\right]$ **1**

$= \frac{\sqrt{3}}{8}$ [cos 70° + cos 110° + cos 30°]

$= \frac{\sqrt{3}}{8}$ [cos 70° + cos (180 – 70)° + cos 30°] **1**

$= \frac{\sqrt{3}}{8}$ [cos 70° – cos 70° + $\frac{\sqrt{3}}{2}$]

$= \frac{3}{16}$ = R.H.S. **Hence proved. 1**

8. A = Number of students who take tea
B = Number of students who take coffee
$\therefore$ According to question, **1**

$$n(A) = 26,\ n(B) = 18$$

and $$n(\bar{A} \cap \bar{B}) = 8$$ **1**

But, $$n(\bar{A} \cap \bar{B}) = n(\overline{A \cup B}) = 8$$

So, $$n(A \cup B) = n(\cup) - n(\overline{A \cup B})$$
$$= 40 - 8 = 32$$ **1**

Using $$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$\therefore$ $$32 = 26 + 18 - n(A \cap B)$$
$\Rightarrow$ $$n(A \cap B) = 44 - 32 = 12$$ **1**

9. $$\tan^2\theta + (1-\sqrt{3})\tan\theta - \sqrt{3} = 0$$

It is quadratic equation in $\tan\theta$, so,

$$D = (1-\sqrt{3})^2 + 4 \times 1 \times \sqrt{3}$$
$$= 1 + 3 - 2\sqrt{3} + 4\sqrt{3}$$
$$D = (1+\sqrt{3})^2$$ **1**

$\therefore$ $$\tan\theta = \frac{-(1-\sqrt{3}) \pm \sqrt{(1+\sqrt{3})^2}}{2 \times 1}$$
$$\tan\theta = \frac{(\sqrt{3}-1) \pm (\sqrt{3}+1)}{2}$$ **1**

Taking + ve sign, we get

$$\tan\theta = \sqrt{3} = \tan\frac{\pi}{3}$$

$\Rightarrow$ $$\theta = n\pi + \frac{\pi}{3},\ n \in I$$ **1**

On taking – ve sign, we get

$$\tan\theta = -1 = -\tan\frac{\pi}{4} = \tan\left(-\frac{\pi}{4}\right)$$

$\Rightarrow$ $$\theta = n\pi - \frac{\pi}{4},\ n \in I.$$ **1**

10. We have a word *EQUATION* consists 8 letters with *A, E, I, O, U* vowels. **1**
We can chose one vowel out of 5 in 5 ways and remaining letters can arrange in 7 ! ways **2**

$\therefore$ Total words starting with vowel $= 5 \times 7\,!$
$= 5 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ **1**
$= 25200.$

OR

Given that,

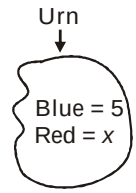
$$P(n, 4) : P(n-1, 3) = 9 : 1$$

$\Rightarrow$ $$\frac{n\,!}{(n-4)\,!} \times \frac{(n-4)\,!}{(n-1)\,!} = \frac{9}{1}$$ **2**

$\Rightarrow$ $$\frac{n}{1} = 9$$

$\Rightarrow$ $$\boxed{n = 9}$$ **2**

11. An Urn consists 5 blue and x red balls.



$\therefore$ Total balls = $5 + x$ **1**

Probability of drawing two blue balls is $\frac{5}{14}$

$$\therefore \quad \frac{5}{14} = \frac{{}^5C_2}{{}^{5+x}C_2} \qquad \textbf{1}$$

$$\Rightarrow \quad \frac{5}{14} = \frac{5 \times 4}{(5+x)(4+x)}$$

$$\Rightarrow \quad x^2 + 9x + 20 = 56 \qquad \textbf{1}$$

$$\Rightarrow \quad x^2 + 9x - 36 = 0$$

$$\Rightarrow \quad (x + 12)\ (x - 3) = 0$$

$$\Rightarrow \quad x = 3,\ x = -12 \text{ (not possible)}$$

$$\therefore \quad \boxed{x = 3} \qquad \textbf{1}$$

12.

$$\lim_{x\to 1} \frac{\left(1 - \frac{1}{x}\right)}{\sin[\pi(x-1)]} = \lim_{x\to 1} \frac{(x-1)\times\pi}{\pi \times x \sin \pi(x-1)} \qquad \textbf{1}$$

$$= \lim_{x\to 1} \frac{1}{\pi x} \times \lim_{x\to 1}\left(\frac{\pi(x-1)}{\sin \pi(x-1)}\right) \qquad \textbf{2}$$

$$= \frac{1}{\pi} \times 1 = \frac{1}{\pi} \qquad \left[\because \lim_{x\to 0}\left(\frac{x}{\sin x}\right) = 1\right] \qquad \textbf{1}$$

OR

$$\lim_{x\to 0} \frac{\tan 8x}{\sin 3x} = \lim_{x\to 0}\left[\frac{\left(\frac{\tan 8x}{8x}\right)\times 8x}{\left(\frac{\sin 3x}{3x}\right)\times 3x}\right] \qquad \textbf{2}$$

$$= \frac{8}{3}\left[\frac{\lim_{x\to 0}\left(\frac{\tan 8x}{8x}\right)}{\lim_{x\to 0}\left(\frac{\sin 3x}{3x}\right)}\right] \qquad \textbf{1}$$

$$= \frac{8/3 \times 1}{1} = \frac{8}{3} \qquad \left(\because \lim_{x\to 0}\frac{\sin x}{x} = 1,\ \lim_{x\to 0}\frac{\tan x}{x} = 1\right) \qquad \textbf{1}$$

13. Given that,

$$x + iy = \sqrt{\frac{a+ib}{p+iq}}$$

$$\Rightarrow \quad (x - iy) = \sqrt{\frac{a-ib}{p-iq}} \qquad \textbf{1}$$

$$\therefore \quad (x + iy)\ (x - iy) = \sqrt{\frac{a+ib}{p+iq}} \times \sqrt{\frac{a-ib}{p-iq}} \qquad \textbf{2}$$

$$x^2 + y^2 = \sqrt{\left(\frac{a^2+b^2}{p^2+q^2}\right)}$$

$\Rightarrow$ $$(x^2 + y^2)^2 = \left(\frac{a^2+b^2}{p^2+q^2}\right)$$ **1**

Hence proved.

OR

Let $$x + iy = \sqrt{-16+30i}$$

Squaring both sides,

$$x^2 - y^2 + 2i\,xy = -16 + 30\,i$$

$\Rightarrow$ $x^2 - y^2 = -16$ and $2xy = 30$ ....(i)

$\therefore$ $$(x^2 + y^2) = (x^2 - y^2)^2 + (2xy)^2$$
$$= 256 + 900$$
$$= 1156$$

$\Rightarrow$ $x^2 + y^2 = 34$ ....(ii) **1**

On solving equations (i) and (ii), we get

$x^2 = 9$ and $y^2 = 25$

$\Rightarrow$ $x = \pm 3$ and $y = \pm 5$

As $2xy$ is positive. So,

$$\sqrt{-16+30i} = \pm(3 + 5i)$$ **1**

Similarly take

$$u + iv = \sqrt{-16-30i}$$

$\Rightarrow$ $u^2 - v^2 + 2iuv = -16 - 30\,i$

$\Rightarrow$ $u^2 - v^2 = -16,\ 2uv = -30$ ...(iii)

$\Rightarrow$ $$(u^2 + v^2)^2 = (-16)^2 + (-30)^2$$
$$= 1156$$

$u^2 + v^2 = 34$ ....(iv) **1**

On solving (iii) and (iv),

$u^2 = 9$ and $v^2 = 25$

$\Rightarrow$ $u = \pm 3$ and $v = \pm 5$

As $2uv$ is negative.

$\therefore$ $$\sqrt{-16-30i} = \pm(3 - 5i)$$

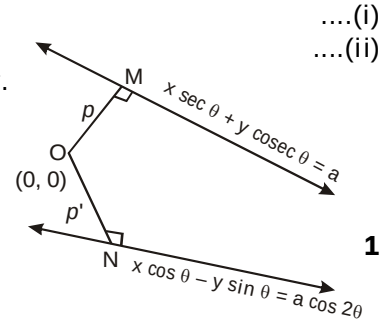
Hence $$\sqrt{-16+30i} + \sqrt{-16-30i} = \pm(3 + 5i) \pm (3 - 5i)$$
$$= \pm 6.$$ **1**

14. We have two equations,

$x \sec\theta + y \operatorname{cosec}\theta = a$ ....(i)

and $x \cos\theta - y \sin\theta = a \cos 2\theta$ ....(ii)

p and p' be $\perp$ distances from O (0, 0) to line (i) and (ii) respectively.



So, $$OM = \left|\frac{0\cdot\sec\theta + 0\cdot\operatorname{cosec}\theta - a}{\sqrt{\sec^2\theta + \cos^2\theta}}\right|$$

$\therefore$ $$p = \left|\frac{-a}{\sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}}\right|$$ **1**

$\Rightarrow \qquad p^2 = \dfrac{a^2.\sin^2\theta.\cos^2\theta}{(\sin^2\theta+\cos^2\theta)}$

$\Rightarrow \qquad p^2 = a^2 \sin^2\theta \cos^2\theta$ ....(i)

Now $\qquad ON = p' = \left|\dfrac{0-0-a\cos 2\theta}{\sqrt{\cos^2\theta+\sin^2\theta}}\right|$

$\Rightarrow \qquad (p')^2 = a^2 \cos^2 2\theta$ ....(ii) **1**

From (i) and (ii),

$$4p^2 + p'^2 = 4a^2 \sin^2\theta \cos^2\theta + a^2 \cos^2 2\theta$$
$$= a^2 \sin^2 2\theta + a^2 \cos^2 2\theta$$
$$= a^2 (\sin^2 2\theta + \cos^2 2\theta)$$
$$= a^2$$ **2**

$\Rightarrow \qquad \boxed{4p^2 + p'^2 = a^2}$ **Hence proved.**

15. There are 52 playing cards.

Total ways of drawing two cards = ${}^{52}C_2$

A = Two cards drawn are red.

B = Two cards drawn are kings

$\therefore \qquad P(A) = \dfrac{{}^{26}C_2}{{}^{52}C_2} = \dfrac{26\times 25}{52\times 51} = \dfrac{25\times 26}{52\times 51}$ **1**

$P(B) = \dfrac{{}^{4}C_2}{{}^{52}C_2} = \dfrac{4\times 3}{52\times 51} = \dfrac{12}{52\times 51}$ **1**

and $\qquad P(A \cap B) = \dfrac{{}^{2}C_2}{{}^{52}C_2} = \dfrac{2\times 1}{52\times 51} = \dfrac{2}{52\times 51}$ **1**

$\therefore \qquad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{25\times 26}{52\times 51} + \frac{12}{52\times 51} - \frac{2}{52\times 51}$$

$$= \frac{2[325+6-1]}{52\times 51}$$

$$= \frac{2\times 330}{52\times 51} = \frac{55}{221}$$

$\therefore \qquad P(A \cup B) = \dfrac{55}{221}$ **1**

16.

C.I.	***f***	**C.F.**	**C.I.**	x_i	$\lvert x_i - \text{Med}\rvert$	$f_i \lvert x_i - \text{Med}\rvert$
16 – 20	5	5	15·5 – 20·5	18	20	100
21 – 25	6	11	20·5 – 25·5	23	15	90
26 – 30	12	23	25·5 – 30·5	28	10	120
31 – 35	14	37	30·5 – 35·5	33	5	70
36 – 40	26	63	35·5 – 40·5	38	0	0
41 – 45	12	75	40·5 – 45·5	43	5	60
46 – 50	16	91	45·5 – 50·5	48	10	160
51 – 55	9	100	50·5 – 55·5	53	15	135
Total	**100**					**735**

$$N = 100, \ \frac{N}{2} = 50$$ **1**

$$l = 35{\cdot}5, \ f = 26, \ cf = 37$$

$\therefore$ $$\text{Median} = 35{\cdot}5 + \left(\frac{50-37}{26}\right) \times 5$$

$$= 35{\cdot}5 + \frac{5}{2}$$

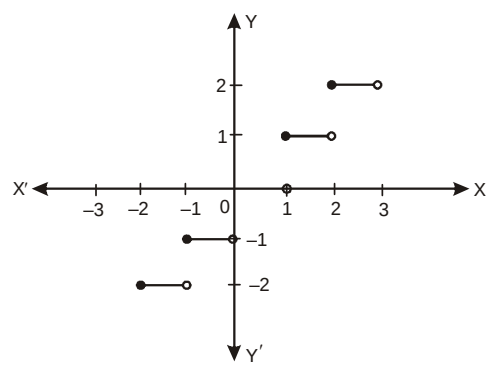
$$\text{Median} = 38$$ **1**

$\therefore$ Mean deviation from median

$$= \frac{\Sigma f_i \, | x_i - \text{Med} |}{\Sigma f_i} = \frac{735}{100} = 7{\cdot}35$$

$$\boxed{\text{M.D. (from median)} = 7{\cdot}35}$$ **2**

17. We have a greatest integer function $f(x) = [x]$, is less than or equal to x



$f(0) = [0] = 0$ **1**

$f(\cdot 5) = [\cdot 5] = 0$

$f(1) = [1] = 1$

$f[-\cdot 5] = [-\cdot 5] = -1$ **1**

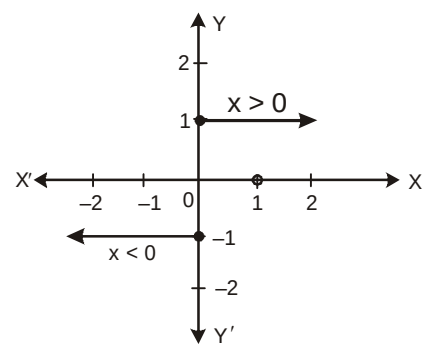
$f[-1] = [-1] = -1$

$f[-1{\cdot}7] = [-1{\cdot}7] = -2$ **2**

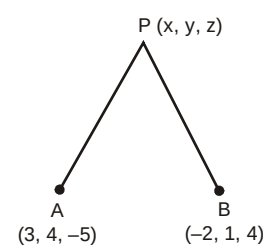
OR

Signum function $f : R \to R$ is

$$f(x) = \begin{cases} 1 \ ; \ x > 0 \\ 0 \ ; \ x = 0 \\ -1 \ ; \ x < 0 \end{cases}$$



18. It is given that



$$PA = PB$$

$\Rightarrow$ $$PA^2 = PB^2$$

$\Rightarrow$ $(x-3)^2 + (y-4)^2 + (z+5)^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$ **1**

$\Rightarrow$ $x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 + 10z$

$= x^2 + 4 + 4x + y^2 + 1 - 2y + z^2 + 16 - 8z$ **2**

$\Rightarrow$ $-6x - 4x - 8y + 2y + 10z + 8z + 50 - 21 = 0$

$\Rightarrow$ $-10x - 6y + 18z + 29 = 0$

$\Rightarrow$ $10x + 6y - 18z - 29 = 0$ **1**

19. $$\text{L.H.S.} = \sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}}$$

$$= \sqrt{2 + \sqrt{2 + \sqrt{2 + (1 + \cos 8\theta)}}}$$ **1**

But $1 + \cos 2A = 2\cos^2 A$

$\therefore \quad \sqrt{2+\sqrt{2+\sqrt{4\cos^2 4\theta}}} = \sqrt{2+\sqrt{2+2\cos 4\theta}}$ **2**

$= \sqrt{2+\sqrt{4\cos^2 2\theta}} = \sqrt{2(1+\cos 2\theta)}$

$= 2\cos\theta$ = R.H.S. **1**

Section - C

20. Four lines (given) forms a parallelogram.

$\sqrt{3}x + y = 0$ ....(i)

$\sqrt{3}y + x = 0$ ....(ii)

$\sqrt{3}x + y = 1$ ....(iii)

$\sqrt{3}y + x = 1$ ....(iv)

On solving (i) and (ii),

$x = 0$ and $y = 0$, *i.e.,* A (0, 0) **1**

Solving (ii) and (iii), we get

$-\sqrt{3}(\sqrt{3}y) + y = 1$

$\Rightarrow \quad -2y = 1 \Rightarrow y = \frac{-1}{2}$

and $\quad x = -\sqrt{3} \times \left(-\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$

$\therefore \quad B\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$ **1**

Solving equations (iii) and (iv), we get

$\sqrt{3}(1-\sqrt{3}x) + x = 1$

$\Rightarrow \quad \sqrt{3} - 2x = 1$

$x = \frac{\sqrt{3}-1}{2}$

and $\quad y = \sqrt{3}\left(\frac{\sqrt{3}-1}{2}\right) + 1$

$= \frac{-3+2+\sqrt{3}}{2} = \frac{\sqrt{3}-1}{2}$

i.e., $C\left(\frac{\sqrt{3}-1}{2}, \frac{\sqrt{3}-1}{2}\right)$ **1**

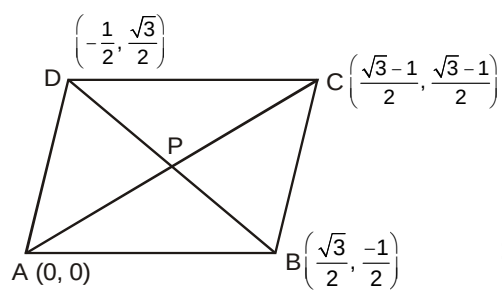
Now, and on solving (i) and (iv),

$\sqrt{3}(-\sqrt{3}x) + x = 1$

$-2x = 1$

$\Rightarrow \quad x = -\frac{1}{2}$

$\Rightarrow \quad y = -\sqrt{3} \times \left(-\frac{1}{2}\right)$



Hence, $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **1**

$\therefore \quad$ Slope of $AC = \left(\frac{\sqrt{3}-1}{2}\right) \div \left(\frac{\sqrt{3}-1}{2}\right) = 1$

and $$\text{slope of } BD = \frac{\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)}{\left(\frac{13}{2}+\frac{1}{2}\right)} = -1$$ **1**

$\therefore$ Product of slopes of AC and BD

$$= 1 \times (-1) = -1$$

Hence, diagonals of $ABCD$ are $\perp$ to each other. **1**

21. $$\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6$$

$$= (\sqrt{2})^6 + {}^6C_1(\sqrt{2})^5 + {}^6C_2(\sqrt{2})^4 + {}^6C_3(\sqrt{2})^3$$

$$+ {}^6C_4(\sqrt{2})^2 + {}^6C_5(\sqrt{2})^1 + {}^6C_6 + (\sqrt{2})^6 - {}^6C_1(\sqrt{2})^5 + {}^6C_2(\sqrt{2})^4$$

$$- {}^6C_3(\sqrt{2})^3 + {}^6C_4(\sqrt{2})^2 - {}^6C_5(\sqrt{2}) + {}^6C_6$$ **2**

$$= 2\left[(\sqrt{2})^6 + {}^6C_2(\sqrt{2})^4 + {}^6C_4(\sqrt{2})^2 + {}^6C_6\right]$$ **1**

$$= 2\left[8 + \frac{6\times 5}{2\times 1}\times 4 + \frac{6\times 5}{2\times 1}\times 2 + 1\right]$$ **1**

$$= 2\,[8 + 60 + 30 + 1]$$ **1**

$$= 2 \times 99 = 198.$$ **1**

OR

We have an expansion $\left(2x^2 - \frac{3}{x}\right)^{11}$

$\therefore$ $$T_{r+1} = (-1)^r\, {}^{11}C_r (2x^2)^{11-r}\left(\frac{3}{x}\right)^r$$ **2**

$$= (-1)^r . {}^{11}C_r\, 2^{11-r} . r^{22-2r} . 3^r . x^{-r}$$

$$= (-1)^r . {}^{11}C_r . 2^{11-r} . 3^r . x^{22-3r}$$ **2**

For term involving x^5 $\quad 22 - 3r = 5$

$\Rightarrow$ $$r = \frac{17}{3} \text{ (which is not an integer)}$$ **2**

$\therefore$ This expansion does not include any term consisting x^5 .

22. (i) $$\text{R.H.S.} = \frac{b^2 - c^2}{a^2}$$

By sine rule, we know that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{C}{\sin C} = k \text{ (say)}$$ **1**

$\therefore$ $a = k \sin A$, $b = k \sin B$, $c = k \sin C$

$\Rightarrow$ $$\frac{\sin^2 B - \sin^2 C}{\sin^2 A}$$ **1**

But $\quad A + B + C = \pi \;\Rightarrow\; A = \pi - (B + C)$

$\Rightarrow$ $$\frac{(\sin B + \sin C)(\sin B - \sin C)}{\sin^2[\pi - (B + C)]}$$

$$= \frac{2\sin\left(\frac{B+C}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)\cdot 2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)}{\sin^2(B+C)}$$ **2**

$$= \frac{\left[2\sin\left(\frac{(B+C)}{2}\right)\cdot\cos\left(\frac{B+C}{2}\right)\right]\cdot\left[2\sin\left(\frac{B-C}{2}\right).\cos\left(\frac{B-C}{2}\right)\right]}{\sin^2(B+C)}$$ **1**

$$= \frac{\sin(B+C)\cdot\sin(B-C)}{\sin^2(B+C)}$$

$$= \frac{\sin(B-C)}{\sin(B+C)} = \text{R.H.S.}$$ **Hence proved**

(ii) L.H.S.= $b \cos B + c \cos C$

Using sine rule, we get

$b = k \sin B$ and $c = k \sin C$ **1**

$\therefore$ $k \sin B \cos B + k \sin C \cos C$ **1**

$= \frac{k}{2}(2 \sin B \cos B + 2 \sin C \cos C)$ **1**

$= \frac{k}{2}[\sin 2B + \sin 2C]$

$= \frac{k}{2}[2 \sin (B + C).\cos (B - C)]$ **1**

$= k \sin (B + C) \cos (B - C)$

$= k \sin (\pi - A) \cos (B - C)$ $(\because B + C = \pi - A)$ **1**

$= k \sin A \cos (B - C)$

$= a \cos (B - C) = \text{R.H.S.}$ **Hence proved. 1**

23. An aged person writes letters to 4 of his friends and ask to copy of letter mailed to 4 people and with same instruction. **1**

So, sequence of letters (numbers) is

$4, 4^2, 4^3, 4^4, \ldots\ldots\ldots\ldots$ **1**

which is G.P. with $a = 4$ and $r = 4$.

So, number of mailed letter up to 8th mail.

= sum of 8 terms of this G.P.

$$= \frac{4(4^8 - 1)}{(4-1)}$$ **2**

$$= \frac{4}{3}\times(256\times 256 - 1)$$

$$= \frac{4}{3}\times(65536 - 1)$$

$$= \frac{4}{3}\times 65535 = 87380$$

Cost of mailing to each letter = Rs. (0·5).

So, total cost = 87380 × 0·5

= Rs. 43690. **1**

Importances :

(i) The ancient means was so popular, effective and people wrote and share all emotional feellings and wait for the response.

(ii) Modern means works activity and we don't feel distances among relationship. **1**

24. $2x + y \geq -3$ ....(i)

$2x + y \leq 6,\ x \geq 0,\ y \geq 0$ ....(ii)

Take $2x + y = -3$

At $x = 0$, $y = -3$

and at $y = 0$, $x = -3/2$ **1**

At (0, 0), $0 + 0 > -3$ (True)

So origin is included. **1**

Now take $2x + y = 6$

At $x = 0$, $y = 6$

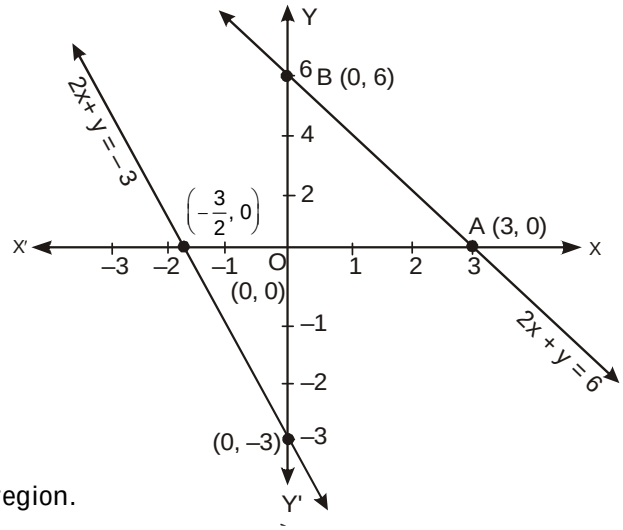
and at $y = 0$, $x = 3$

At (0, 0), $0 + 0 < 6$ (True) **1**

So here origin is also included

$x \geq 0,\ y \geq 0$ mean first quadrant region. **2**

The shaded region *OAB* is the solution region. **1**



25. $P(n) = 5^{2n} - 1$ is divisible by 24.

Put $n = 1$.

$P(1) : 5^2 - 1 = 24$, it is divisible by 24

i.e., $P(1)$ is true. **1**

Let $P(k)$ is true.

$\therefore$ $P(k) = 5^{2k} - 1 = 24\lambda$; $(\lambda \in N)$ ...(i) **2**

Now to prove that $P(k + 1)$ is true.

i.e., $5^{2(k+1)} - 1$ is divisible by 24.

Take L.H.S. $= 5^{2(k+1)} - 1$

$= 5^{2k}.\ 5^2 - 1$

$= 25.(24\lambda + 1) - 1$ [using equation (i)] **2**

$= 25 \times 24\lambda + 25 - 1$

$= 24\ (25\lambda + 1)$

$= 24\mu$; $(\mu = 25\lambda + 1 \in N)$

i.e., $P(k + 1)$ is true. **1**

Hence, $P(n)$ is true when $P(k)$ is true for all $n \in N$

26. There are 3 Mathematics, 4 Hindi and 5 physics books.

So, Total books = 12 **1**

$\therefore$ Total arrangement of these books in a shelf

= 12 ! ways **2**

When all books of same subject together then these can be arranged in 3 !, 4 ! and 5 ! ways respectively. **1**

$\therefore$ Total ways of such arragements $= 3! \times 3! \times 4! \times 5!$

$= 6 \times 6 \times 24 \times 120$

$= 1,03,680.$ **1**

The students, who have such habbits of well and proper arrangements of things can be good in behaviour, and disciplined alos. **1**

❑❑

SAMPLE QUESTION PAPER–9

Section-A

1. $A = \{a, b, c\}$

i.e., $n(A) = 3$

$\therefore$ Number of subsets $= 2^3 = 8.$ **1**

2. Equation of line is $3x - 4y + 9 = 0$

$\therefore$ Equation of $\perp$ line is $4x + 3y + \lambda = 0$ **1**

3. ${}^{16}C_r = {}^{16}C_{r+2}$ (given)

$\Rightarrow$ $r + r + 2 = 16 \quad \Rightarrow \quad r = 7$

$\therefore$ ${}^{7}C_4 = \dfrac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1} = 35.$ **1**

4. Equation of circle is $x^2 + y^2 - 4x + 6y - 12 = 0$

$\therefore$ $C\ (2, -3)$

$R = \sqrt{2^2 + (-3)^2 + 12} = 5$ **1**

5. $\dfrac{d}{dx}\log_a x = \dfrac{1}{x}.\log_a e$ **1**

6. Parabola is $2x^2 + y = 0 \Rightarrow x^2 = -\dfrac{1}{2}y$

$\therefore$ Focus $\left(0, -\dfrac{1}{8}\right)$, Directrix $= 8y - 1 = 0$ **½ + ½**

Section-B

7. $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 3\}$, $C = \{5\}$.

$\therefore$ $(B \cap C) = \{\ \}$ **1**

$\Rightarrow$ $A - (B \cap C) = \{1, 2, 3, 4, 5\} - \{\ \}$

$= \{1, 2, 3, 4, 5\} =$ L.H.S. **1**

and $(A - B) = \{1, 2, 3, 4, 5\} - \{2, 3\}$

$= \{1, 4, 5\}$

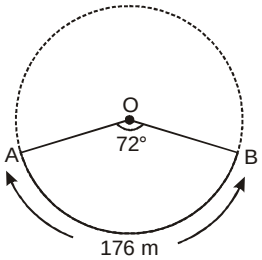
and $(A - C) = \{1, 2, 3, 4, 5\} - \{5\}$

$= \{1, 2, 3, 4\}$ **1**

$\Rightarrow$ $(A - B) \cup (A - C) = \{1, 2, 3, 4, 5\} =$ R.H.S. **1**

$\therefore$ L.H.S. = R.H.S. **Hence verified.**

8. A horse in tied to a point at O, $OA = OB$, (Rope) horse moves along a circular path keeping rope tight. **2**



$\therefore$ Length of rope $= \dfrac{\text{Arc } AB}{\angle AOB} = \dfrac{176 \times 180°}{72 \times \pi}$ **1**

Length of rope $= \dfrac{176 \times 180 \times 7}{72 \times 22} = 180$ m. **1**

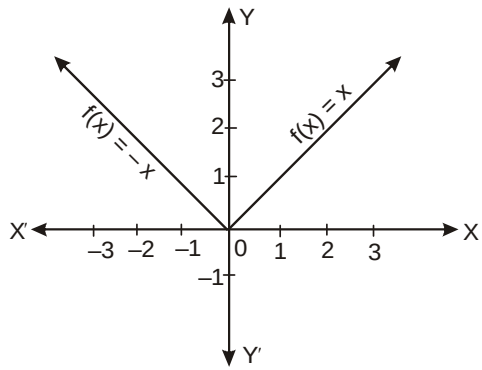
9. **Modulus function :** A function $f : R \to R$ is defined as

$$f(x) = |x| \ \forall\ x \in R$$

$\therefore$ $$f(x) = |x| = \begin{cases} x: & x \geq 0 \\ -x: & x < 0 \end{cases}$$ **1**

i.e., Absolute/modulus value of any function is always positive. **1**

Graph :



2

OR

Given, $f(x) = ax + b$

$f(-1) = -5$ **1**

$\Rightarrow$ $-a + b = -5$ ....(i) **1**

and $f(3) = 3$

$\Rightarrow$ $3a + b = 3$ ....(ii) **1**

On solving (i) and (ii), we get

$$\boxed{a = 2,\ b = -3}$$ **1**

10. $\cos\theta + \sin\theta = \sqrt{2}\ \cos\theta,$ (given)

$\Rightarrow$ $\sin\theta = \sqrt{2}\cos\theta - \cos\theta$ **1**

$\Rightarrow$ $\sin\theta = (\sqrt{2} - 1)\ \cos\theta$

$\Rightarrow$ $\dfrac{\sin\theta}{\sqrt{2}-1} = \cos\theta$ **1**

On rationalising the denominator

$$\frac{\sin\theta(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \cos\theta$$ **1**

$\Rightarrow$ $\sqrt{2}\sin\theta + \sin\theta = \cos\theta$

$\Rightarrow$ $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$ **Hence proved.** **1**

OR

If $\tan\alpha = \dfrac{m}{m+1}$ and $\tan\beta = \dfrac{1}{2m+1}$

$\therefore$ $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta}$

$\Rightarrow$ $$\tan(\alpha + \beta) = \frac{\left(\dfrac{m}{m+1} + \dfrac{1}{2m+1}\right)}{\left(1 - \dfrac{m}{m+1} \times \dfrac{1}{2m+1}\right)}$$ **1**

$$= \frac{\left(\frac{m(2m+1)+(m+1)}{(m+1)(2m+1)}\right)}{\left(\frac{(m+1)\,(2m+1)-m}{(m+1)(2m+1)}\right)}$$ **1**

$$= \left(\frac{2m^2+m+m+1}{2m^2+2m+m+1-m}\right)$$ **1**

$$= \frac{2m^2+m+1}{2m^2+2m+1}$$

$$\tan(\alpha+\beta) = 1 = \tan\frac{\pi}{4}$$ **1**

$\Rightarrow$ $$\alpha+\beta = \frac{\pi}{4}$$

11. $$P(n) : 1 + 4 + 7 + ... + (3n-2) = \frac{1}{2}n(3n-1)$$

Put $n = 1$

$$P(1) : 1 = \frac{1}{2}(3+1)$$

$\Rightarrow$ $$1 = 1$$

So $P(1)$ is true.

Let $P(k)$ is true **½**

i.e., $$P(k) : 1 + 4 + 7 + ... + (3k-2) = \frac{1}{2}k(3k-1) \quad ...(i)$$

Now to prove $P(k+1)$ is true.

i.e., $$1 + 4 + 7 + ... + (3k-2) + [3k+1-2] = \frac{1}{2}(k+1)[3(k+1)-1]$$ **½**

Taking L.H.S.

$$1 + 4 + 7 + ... + (3k-2) + (3k+3-2)$$ **1**

Using equation (i), we get

$$\frac{1}{2}k(3k-1)+(3k+1) = \frac{1}{2}\left[3k^2-k+6k+2\right]$$

$$= \frac{1}{2}\left[3k^2+5k+2\right]$$ **1**

$$= \frac{1}{2}(3k^2+3k+2k+2)$$

$$= \frac{1}{2}[3k(k+1)+2(k+1)]$$ **½**

$$= \frac{1}{2}[(3k+2)(k+1)]$$

$$= \frac{1}{2}(k+1)[3(k+1)-1]$$ **½**

= R.H.S. **Hence proved.**

12.

Class	f	x	$u_i = \dfrac{x_i - 45{\cdot}5}{6}$	u_i^2	$f_i u_i^2$	$f_i u_i$
30·5 – 36·5	4	33·5	–2	4	16	–8
36·5 – 42·5	10	39·5	–1	1	10	–10
42·5 – 48·5	14	45·5(a)	0	0	0	0
48·5 – 54·5	27	51·5	1	1	27	27
54·5 – 60·5	45	57·5	2	4	180	90
Total	**100**				**233**	**99**

Here, $a = 45{\cdot}5$, $h = 6$, $N = 100$ **2**

$$\text{Variance} = h^2\left[\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2\right]$$

$$= 36\left[\frac{233}{100} - \left(\frac{99}{100}\right)^2\right]$$ **1**

$$= 36\,[2{\cdot}33 - ({\cdot}99)^2\,]$$

$$= 36\,[2{\cdot}33 - 0{\cdot}980]$$

$$= 36 \times 1{\cdot}35$$

$$\text{Variance} = 48{\cdot}6.$$ **1**

13. An urn contains 1 red and 1 black ball, a ball is drawn, if gets red toss a coin. If gets black throw a dice, then

$S = \{RH, RT, B_1, B_2, B_3, B_4, B_5, B_6\}$

A = Event of getting an even number *i.e.,* (B_2, B_4, B_6) **1**

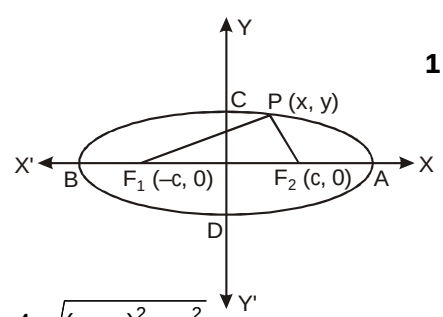
$$\therefore \quad P(A) = \frac{\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{6}}{\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}+\frac{1}{6}\times\frac{1}{2}\times 6}$$ **1**

$$= \frac{\left(\frac{3}{12}\right)}{\left(\frac{1}{2}+\frac{1}{2}\right)}$$

$$P(A) = \left(\frac{\frac{1}{4}}{1}\right) = \frac{1}{4}.$$ **1**

14. According to definition of ellipse

$$PF_1 + PF_2 = 2a \text{ (constant)}$$ **1**



$$\Rightarrow \quad \sqrt{(x-c)^2+y^2} + \sqrt{(x+c)^2+y^2} = 2a$$

$$\Rightarrow \quad \sqrt{(x+c)^2+y^2} = 2a - \sqrt{(x-c)^2+y^2}$$

Squaring both sides, we get

$$x^2 + c^2 + 2cx + y^2 = 4a^2 + x^2 + c^2 - 2cx + y^2 - 4a\sqrt{(x-c)^2+y^2}$$

$\Rightarrow \quad 4cx - 4a^2 = -4a\sqrt{(x-c)^2 + y^2}$

$\Rightarrow \quad (cx - a^2) = -a\sqrt{(x-c)^2 + y^2}$ **1**

Again squaring both sides, we get

$$c^2x^2 + a^4 - 2a^2cx = a^2(x^2 + c^2 - 2cx + y^2)$$

$\Rightarrow \quad c^2x^2 + a^4 - 2a^2cx = a^2x^2 + a^2c^2 - 2a^2cx + a^2y^2$

$\Rightarrow \quad (a^2 - c^2)x^2 + a^2y^2 = a^4 - a^2c^2$ **1**

$\Rightarrow \quad (a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$

$\Rightarrow \quad \dfrac{x^2}{a^2} + \dfrac{y^2}{a^2 - c^2} = 1$

$\Rightarrow \quad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \qquad (\because a^2 - c^2 = b^2)$ **1**

It is standard equation of ellipse.

15. We have 600 litres of a 12% acidic solution.

So total acid = $\dfrac{600 \times 12}{100}$ litre.

Let x litres solution of 30% is to be added. Total mixture = $(600 + x)$ litres. **1**

So, according to question,

$$\frac{15}{100}(600 + x) < \frac{600 \times 12}{100} + \frac{x \times 30}{100} < \frac{18(600 + x)}{100}$$ **1**

Take $\quad \dfrac{15}{100}(600 + x) < \dfrac{7200 + 30x}{100}$

$\Rightarrow \quad 9000 + 15x < 7200 + 30x$

$\Rightarrow \quad 15x > 1800$

$\Rightarrow \quad x > 120$

Now take $\quad \dfrac{600 \times 12}{100} + \dfrac{x \times 30}{100} < \dfrac{18}{100}(600 + x)$

$\Rightarrow \quad 7200 + 30x < 10800 + 18x$ **1**

$12x < 3600$

$x < 300$

i.e., $\quad 120 < n < 300$ **1**

So we have to mix 30% acid solution between 120 litres to 300 litres.

OR

$\therefore$ Linear inequations are

$2x + y > 4 \quad$ ....(i)

$x + y < 3 \quad$ ....(ii)

and $\quad 2x - 3y < 6 \quad$ ....(iii) **½**

Take $\quad 2x + y = 4$

At $x = 0$, $\quad y = 4$

and at $y = 0$, $\quad x = 2$

At (0, 0), $\quad 0 + 0 > 4$ (False) **½**

i.e., origin is not included.

Now take $x + y = 3$

At $x = 0$, $y = 3$ and

At $y = 0$, $x = 3$

At (0, 0), $0 + 0 < 3$ (True) **1**

So origin is included.

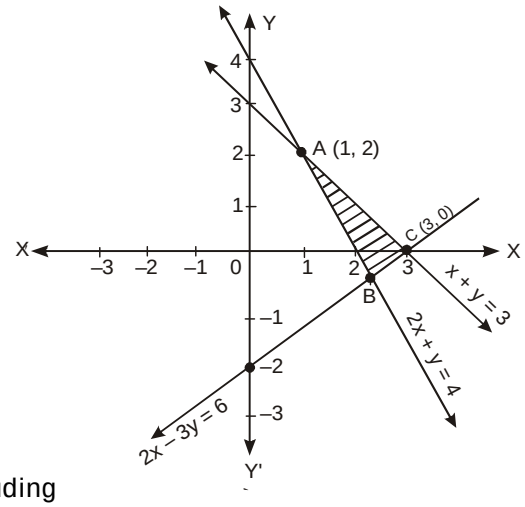
Now take $2x - 3y = 6$

At $x = 0$, $y = -2$

and at $y = 0$, $x = 3$

At (0, 0), $0 - 0 < 6$ (True) **1**

So origin is included. **1**



The shaded region *ABC* is the solution region excluding the lines. All the points within this region will satisty these inequations.

16. Given $a + b + c \neq 0$

and $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P. **1**

$\Rightarrow$ $\frac{b+c}{a}+1, \frac{c+a}{b}+1, \frac{a+b}{c}+1$ are also in A.P. **1**

$\Rightarrow$ $\frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ are in A.P. **1**

$\Rightarrow$ $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P. **1**

17. $$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x - 1$$

$$f'(x) = \frac{100x^{99}}{100} + \frac{99x^{98}}{99} + ... + \frac{2x}{2} + 1$$ **1**

$$f'(x) = x^{99} + x^{98} + ... + x + 1$$ **1**

$\therefore$ $f'(1) = 1 + 1 + 1 + ...$ 100 times $= 100$

and $f'(1) = 0 + 0 + ... + 0 - 1 = 1$ **1**

$\Rightarrow$ $f'(1) = 100 \times f'(0)$ **Hence proved. 1**

18. There is a girder (large metal beam) is in the shape of parabola. **1**

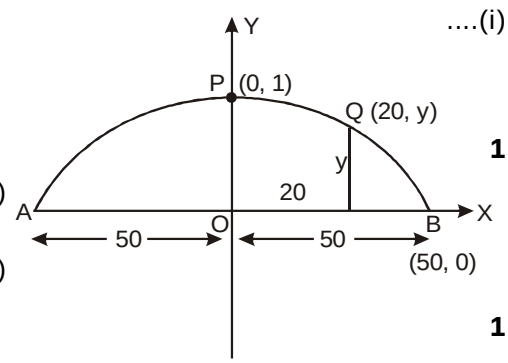
So equation of girder is $x^2 = -4a(y - 10)$ ....(i)

B (50, 0) lies on (i)

$\therefore$ $2500 = -4a(0 - 10)$

$\Rightarrow$ $4a = 250$ **1**

$\Rightarrow$ $x^2 = -250(y - 10)$



Also *Q* (20, *y*) lies on it

$\therefore$ $400 = -250(y - 10)$

$\Rightarrow$ $y - 10 = -\frac{400}{250}$ **1**

$y = 10 - 1{\cdot}6 = 8{\cdot}4$ metre

$\therefore$ Height of girder 20 m from centre is 8·4 metre. **1**

OR

Equation of ellipse $16x^2 + y^2 = 16$

$\Rightarrow$ $$\frac{x^2}{1} + \frac{y^2}{16} = 1$$

$a^2 = 1 \Rightarrow a = \pm 1$ and $b^2 = 16 \Rightarrow b = \pm 4$ **1**

$\because$ $(b > a)$

$\Rightarrow$ Y-axis is axis of ellipse.

$\therefore$ $c^2 = b^2 - a^2 = 16 - 1 = 15$ 1.

$$c = \pm\sqrt{15}$$

$\therefore$ $F(0, \pm\sqrt{15})$, $V(0 \pm 4)$

Length of major axis $= 2b = 2 \times 4 = 8$

Length of minor axis $= 2a = 2 \times 1 = 2$ **1**

$$e = \frac{c}{b} = \frac{\sqrt{15}}{4}$$

and latus-racturm $= \dfrac{2a^2}{b} = \dfrac{2 \times 1 \times 1}{4} = \dfrac{1}{2}$. **1**

19. There are 5 men and 4 women; seated in a row such that woman at even place.

So we have to place all men like

$M_1 \square M_2 \square M_3 \square M_4 \square M_5$ **2**

at $\square$ (boxes) 4 women can sit in 4 ! ways and men in 5 ! ways. **1**

$\therefore$ Total ways of arrangements $= 4! \times 5!$

$= 24 \times 120$

$= 2880.$ **1**

Section-C

20. Quadratic equation is

$$x^2 + (2p + 1)x + p^2 + 2.$$

Let α and β be their roots such that $\beta = 2\alpha$ **1**

$\therefore$ $\alpha + 2\alpha = -(2p + 1)$

$\Rightarrow$ $3\alpha = -(2p + 1)$ ....(i) **1**

and Product of roots $= c/a$

$\alpha.2\alpha = (p^2 + 2)$

$\Rightarrow$ $2\alpha^2 = p^2 + 2$ ....(ii) **1**

On solving equations (i) and (ii),

$$2\left[\frac{-(2p+1)}{3}\right]^2 = p^2 + 2$$ **1**

$\Rightarrow$ $2(4p^2 + 1 + 4p) = 9p^2 + 18$

$\Rightarrow$ $8p^2 + 2 + 8p = 9p^2 + 18$ **1**

$\Rightarrow$ $p^2 - 8p + 16 = 0$

$\Rightarrow$ $(p - 4)^2 = 0$

$\Rightarrow$ $\boxed{p = 4}$ **1**

OR

$$z = x + iy \text{ and } w = \frac{1-iz}{z-i}$$

As $\quad |w| = 1$

$\Rightarrow \quad \left|\frac{1-iz}{z-i}\right| = 1$ **1**

$\Rightarrow \quad \left|\frac{1-i(x+iy)}{(x+iy)-i}\right| = 1$ **1**

$\Rightarrow \quad \left|\frac{(1+y)-ix}{x+i(y-1)}\right| = 1$

$\Rightarrow \quad \sqrt{(1+y)^2+x^2} = \sqrt{x^2+(y-1)^2}$ **2**

$\Rightarrow \quad 1 + y^2 + 2y + x^2 = x^2 + y^2 + 1 - 2y$

$\Rightarrow \quad 4y = 0$ **1**

$\Rightarrow \quad y = 0$

i.e., $\quad z = x + i.0.$

$\Rightarrow$ z is purely real. **1**

21. $\quad$ L.H.S. $= \cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) + \cos^2\left(x+\frac{\pi}{3}\right)$

$$= \left(\frac{1+\cos 2x}{2}\right) + \left(\frac{1+\cos\left(2x+\frac{2\pi}{3}\right)}{2}\right) + \left(\frac{1+\cos\left(2x-\frac{2\pi}{3}\right)}{2}\right)$$ **1**

$$= \frac{3}{2} + \frac{1}{2}\left[\cos 2x + \cos\left(2x+\frac{2\pi}{3}\right) + \cos\left(2x-\frac{2\pi}{3}\right)\right]$$

$$= \frac{3}{2} + \frac{1}{2}\left[\cos 2x + 2\cos(2x) - \cos\left(\frac{2\pi}{3}\right)\right]$$ **1**

$$= \frac{3}{2} + \frac{1}{2}\left[\cos 2x + 2\cos 2x.\cos\left(\pi - \frac{\pi}{3}\right)\right]$$ **1**

$$= \frac{3}{2} + \frac{1}{2}\left[\cos 2x + 2\cos 2x.\left(-\cos\frac{\pi}{3}\right)\right]$$ **1**

$$= \frac{3}{2} + \frac{1}{2}\left[\cos 2x - 2 \times \frac{1}{2}\cos 2x\right]$$ **1**

$$= \frac{3}{2} + 0 = \frac{3}{2} = \text{R.H.S.}$$ **1**

Hence proved.

OR

In $\Delta\, ABC$,

$$\frac{b+c}{12} = \frac{c+a}{13} = \frac{a+b}{15} = k, \text{ (say)}$$

$\Rightarrow$

$$\left.\begin{aligned} b + c &= 12k \\ c + a &= 13k \\ a + b &= 15k \end{aligned}\right\} \quad \text{...(i)}$$ **1**

and

$\Rightarrow \quad 2(a + b + c) = 40k$

$\Rightarrow \quad a + b + c = 20k$ ...(ii)

$\therefore$ Using equations (i) and (ii),

$a = 8k,\ b = 7k$ and $c = 5k$

$\Rightarrow \quad \cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ **1**

$= \dfrac{49k^2 + 25k^2 - 64k^2}{2 \times 7k \times 5k}$

$= \dfrac{10k^2}{70k^2} = \dfrac{1}{7}$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$= \dfrac{64k^2 + 25k^2 - 49k^2}{2 \times 8k \times 5k}$ **1**

$= \dfrac{40k^2}{80k^2} = \dfrac{1}{2}$

and $\cos C = \dfrac{64k^2 + 49k^2 - 25k^2}{2 \times 8k \times 7k}$

$= \dfrac{8k^2}{112k^2} = \dfrac{44}{56}$ **1**

$\therefore \quad \cos A : \cos B : \cos C = \dfrac{1}{7} : \dfrac{1}{2} : \dfrac{44}{56}$

$= \dfrac{16 : 56 : 88}{112}$ **1**

$\Rightarrow \quad \cos A : \cos B : \cos C = 16 : 56 : 88$

$\Rightarrow \quad \cos A : \cos B : \cos C = 2 : 7 : 11$

$\Rightarrow \quad \dfrac{\cos A}{2} = \dfrac{\cos B}{7} = \dfrac{\cos C}{11}$ **1**

Hence proved.

22. Total students in a hostel = 400

A = No. of students who takes tea

B = No. of students who takes milk **1**

$\therefore$ According to question $n(A) = 250,\ n(B) = 200$ **1**

$n(\bar{A} \cap \bar{B}) = 50$ **1**

$\Rightarrow \quad n(\overline{A \cup B}) = 50 \Rightarrow n(A \cup B) = 400 - 50 = 350$ **1**

We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$350 = 250 + 200 - n(A \cap B)$

$\Rightarrow \quad n(A \cap B) = 450 - 350$ **1**

$\Rightarrow \quad n(A \cap B) = 100$

$\therefore$ Number of students who takes both drink = 100. **1**

It seems that mostly students like tea, it shows the traditional drink items are changing in our country.

23. General equation of circle having (h, k) centre and radius r is

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{....(i)}$$

Passing (2, – 2) and (3, 4)

$\Rightarrow$ $$(2-h)^2 + (-2-k)^2 = r^2 \quad \text{....(ii)} \quad \mathbf{1}$$

and $$(3-h)^2 + (4-k)^2 = r^2 \quad \text{....(iii)} \quad \mathbf{1}$$

From (ii) and (iii),

$$(2-h)^2 + (-2-k)^2 = (3-h)^2 + (4-k)^2$$

$\Rightarrow$ $$4 + h^2 - 4h + 4 + k^2 + 4k = 9 + h^2 - 6h + 16 + k^2 - 8k$$

$\Rightarrow$ $$2h + 2k = 17 \quad \text{....(iv)} \quad \mathbf{1}$$

Also (h, k) lies on $x + y = 1$

$\Rightarrow$ $$h + k = 1$$

$\Rightarrow$ $$2h + 2k = 2 \quad \text{....(v)} \quad \mathbf{1}$$

On solving equations (iv) and (v), we get

$$k = \frac{3}{2} \text{ and } y = -\frac{1}{2}$$

From equation (ii), we get

$$\left(2+\frac{1}{2}\right)^2 + \left(-2-\frac{3}{2}\right)^2 = r^2 \quad \mathbf{1}$$

$\Rightarrow$ $$r^2 = \frac{25}{4} + \frac{49}{4} = \frac{74}{4}$$

$\therefore$ From (i), equation of circle is

$$\left(x+\frac{1}{2}\right)^2 + \left(y-\frac{3}{2}\right)^2 = \frac{74}{4}$$

$\Rightarrow$ $$x^2 + y^2 + x - 3y + \frac{1}{4} + \frac{9}{4} = \frac{74}{4}$$

$\Rightarrow$ $$\boxed{x^2 + y^2 + x - 3y - 16 = 0} \quad \mathbf{1}$$

It is required equation of circle.

OR

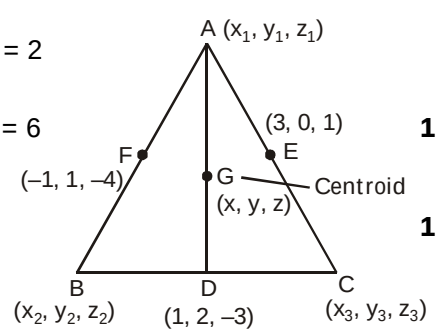
The mid points D, E and F of ΔABC are respectively (1, 2, – 3), (3, 0, 1) and (– 1, 1, – 4).

Using mid point formula,

$$\frac{x_1 + x_2}{2} = -1 \Rightarrow x_1 + x_2 = -2$$

$$\frac{x_2 + x_3}{2} = 1 \Rightarrow x_2 + x_3 = 2$$

and $$\frac{x_3 + x_1}{2} = 3 \Rightarrow x_3 + x_1 = 6 \quad \mathbf{1}$$



On adding all these, we get

$$2(x_1 + x_2 + x_3) = -2 + 2 + 6 = 6 \quad \mathbf{1}$$

$\Rightarrow$ $$x_1 + x_2 + x_3 = \frac{6}{2} = 3$$

$\Rightarrow$ $$\frac{x_1 + x_2 + x_3}{3} = 1 \quad \text{....(i)} \quad \mathbf{1}$$

i.e., $x = \dfrac{x_1 + x_1 + x_3}{3} = 1$ = (x coordinate of centroid)

Now, $\dfrac{y_1 + y_2}{2} = 1 \quad \Rightarrow \quad y_1 + y_2 = 2$

$\dfrac{y_2 + y_3}{2} = 2 \quad \Rightarrow \quad y_2 + y_3 = 4$

and $\dfrac{y_3 + y_1}{2} = 0 \quad \Rightarrow \quad y_3 + y_1 = 0$

$\Rightarrow \quad 2\,(y_1 + y_2 + y_3) = 6$

$\Rightarrow \quad \dfrac{y_1 + y_2 + y_3}{3} = 1$ ....(ii) **1**

Similarly, $\dfrac{z_1 + z_2}{2} = -4 \quad \Rightarrow \quad z_1 + z_2 = -8$

$\dfrac{z_2 + z_3}{2} = -3 \quad \Rightarrow \quad z_2 + z_3 = -6$

and $\dfrac{z_3 + z_1}{2} = 1 \quad \Rightarrow \quad z_3 + z_1 = 2$ **1**

$\Rightarrow \quad 2\,(z_1 + z_2 + z_3) = -12$

$\Rightarrow \quad \dfrac{z_1 + z_2 + z_3}{3} = -2$ ....(iii)

$\therefore$ From equations (i), (ii) and (iii) we get co-ordinates of centroid.

i.e., $G\,(1, 1, -2)$. **1**

24. Binomial expansion is $(x - 7y)^{12}$

$\therefore$ Its general term

i.e., $T_{r+1} = {}^{12}C_r.(x)^{12-r}.\,(-7y)^r$ **1**

$T_{r+1} = (-1)^r.{}^{12}C_r\, x^{12-r}.7^r.\, y^r$ **1**

$= (-1)^r.{}^{12}C_r\, x^{12-r}\; y^r.7^r$

For coeff. of x^5y^7, we need to put $r = 7$ **2**

$\therefore$ Coefficients of $x^5y^7 = (-1)^7\; {}^{12}C_7\; (7)^7$

$= -\dfrac{12\times11\times10\times9\times8}{5\times4\times3\times2\times1}\times7\times7\times7\times7\times7\times7\times7$

$= -65{,}22{,}46{,}056.$ **2**

OR

We have $(x + a)^n$

$T_{r+1} = {}^nC_r\,(x)^{n-r}\,a^r$

3rd, 4th and 5th terms respectively are

${}^nC_2\,(x)^{n-2}\,a^2$, ${}^nC_3\,(x)^{n-3}\,a^3$ and ${}^nC_4\,(x)^{n-4}\,a^4$

$\therefore$ According to question,

${}^nC_2(x)^{n-2}\,a^2 : {}^nC_2\,(x)^{n-3}\,a^3 : {}^nC_4\,(x)^{n-4}.\,a^4$

$= 84 : 280 : 560$

Now take

$$\frac{{}^nC_3 x^{n-3} a^3}{{}^nC_2 (x)^{n-2} a^2} = \frac{280}{84}$$ **1**

$$\frac{n(n-1)(n-2)\times 2}{6\times n(n-1)} \cdot \frac{a}{x} = \frac{70}{21}$$

$\Rightarrow$ $$\frac{n-2}{3} \times \frac{a}{x} = \frac{70}{21}$$

$\Rightarrow$ $(n-2)\,a = 10\,x$ ...(i) **1**

Also take

$$\frac{{}^nC_4 x^{n-4}.a^4}{{}^nC_3\, x^{n-3}\, a^3} = \frac{560}{280}$$

$\Rightarrow$ $$\frac{n(n-1)(n-2)(n-3)\times 6}{24\times n(n-1)(n-2)} \times \frac{a}{x} = 2$$

$\Rightarrow$ $$\frac{n-3}{4} \times \frac{a}{x} = 2$$

$\Rightarrow$ $$\frac{a}{x} = \frac{8}{n-3}$$ ....(ii) **1**

On solving equations (i) and (ii),

$$\frac{10}{n-2} = \frac{8}{n-3}$$

$\Rightarrow$ $10n - 30 = 8n - 16$

$\Rightarrow$ $2n = 14$

$\Rightarrow$ $n = 7$

From (i) we get

$5a = 10x \Rightarrow a = 2x$ ....(iii) **1**

Now using $\quad {}^nC_2\,(x)^{n-2}\,a^2 = 84$

$\Rightarrow$ ${}^7C_2 (x)^{7-2}\,(2n)^2 = 84$

$$\frac{7\times 6}{2\times 1} x^7 \times 4 = 84$$

$\Rightarrow$ $$x^7 = \frac{84}{7\times 6\times 2}$$

$\Rightarrow$ $x^7 = 1 \Rightarrow (1)^7$ **1**

$\Rightarrow$ $x = 1$

$\therefore$ $a = 2x \Rightarrow a = 2$

Hence $x = 1$, $n = 7$ and $a = 2$. **1**

25. There are 12 questions in test it consists two parts say :

Part I consists 5 questions and part II consists 7 questions.

A student must be attend 8 questions, 3 from each.

So total ways of selecting 8, having 3 questions from each part **2**

= {3 in I and 5 in II) or (4 in I and 4 in II) or (5 in I and 3 in II)

$= {}^5C_3 \times {}^7C_5 + {}^7C_4 \times {}^5C_4 + {}^5C_5 \times {}^7C_3$ **2**

$$= \frac{5\times4\times3}{3\times2\times1}\times\frac{7\times6}{2\times1}+\frac{7\times6\times5}{3\times2\times1}+\frac{5\times1}{1}+\frac{5}{5}\times\frac{7\times6\times5}{3\times2\times1}$$

$= 210 + 35 \times 5 + 35$

$= 210 + 175 + 35$

$= 420$ **1**

Teacher can judge the consistency of students through this test, because it is already mentioned that students are unique. **1**

26. We have two A.P.'s say

A.P. I. First term $= a_1$

and common diff. $= d_1$ **½**

and A.P. II. First term $= a_2$

and common diff. $= d_2$

$\therefore$ According to question,

$$\frac{\text{Sum of } n \text{ terms of I A.P.}}{\text{Sum of } n \text{ terms of II A.P.}} = \frac{14-5n}{3n+5}$$ **½**

$$\Rightarrow \quad \frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{14-5n}{3n+5}$$ **1**

$$\Rightarrow \quad \frac{[2a_1+(n-1)d_1]}{[2a_2+(n-1)d_2]} = \frac{14-5n}{3n+5}$$ **1**

For ratio of 8th term of two A.P.'s we have to put

$n - 1 = 14$

$\Rightarrow \quad n = 15$

$$\therefore \quad \frac{[2a_1+14d_1]}{[2a_2+14d_2]} = \frac{14-5\times15}{3\times15+5}$$ **1**

$$\Rightarrow \quad \frac{2[a_1+7d_1]}{2[a_2+7d_2]} = \frac{14-75}{45+5}$$ **1**

$$\Rightarrow \quad \frac{8^{\text{th}} \text{ term of I A.P.}}{8^{\text{th}} \text{ term of II A.P.}} = \frac{-61}{50}.$$ **1**

❑❑

SAMPLE QUESTION PAPER–10

Section-A

1. $A = \phi = \{\ \}$

$\Rightarrow$ $P(A) = 2^0 = 1$ element. **1**

2. $\tan\left(\frac{13\pi}{12}\right) = \tan\left(\pi + \frac{\pi}{12}\right) = +\tan\frac{\pi}{12}$

$\Rightarrow$ $\tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \dfrac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}}$

$= \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$. **1**

3. $\frac{5-2x}{3} \le \frac{x}{6} - 5$

$\Rightarrow$ $10 - 4x \le x - 30$

$\Rightarrow$ $5x \ge 40$

$\Rightarrow$ $x \ge 8 \Rightarrow x = (8, \infty)$. **1**

4. $(x + 3)^8$

$T_{r+1} = {}^8C_r\, x^{8-r}\, 3^r$

$\therefore$ Coefficient of $x^5 = {}^8C_3 \times 3^3 = \frac{8 \times 7 \times 6}{3 \times 2} \times 3 \times 3 \times 3$

$= 56 \times 27 = 1512$. **1**

5. $A\ (-2, 6)$, $B\ (4, 8)$

Slope of $AB = \frac{2}{6} = \frac{1}{3}$

$C\ (8, 12)$ and $D\ (n, 24)$

Slope of $CD = \frac{12}{x-8}$

$\because$ $AB \perp CD \Rightarrow \frac{12}{x-8} \times \frac{1}{3} = -1$

$x - 8 = -4$

$\Rightarrow$ $\boxed{x = 4}$ **1**

6. If two integers a and b are such that $(a - b)$ is always positive integer, then $a > b$. **1**

Section-B

7. L.H.S. : Let $x \in (A \cup B)'$

$\Rightarrow$ $x \notin (A \cup B)$

$\Rightarrow$ $x \notin A$ and $x \notin B$ **½**

$\Rightarrow \quad x \in A'$ and $x \in B'$ **½**

$\Rightarrow \quad x \in A' A \cap B'$

Thus $\quad x \in (A \cup B)' \Rightarrow x \in (A' \cap B')$

$\Rightarrow \quad (A \cup B)' \subseteq (A' \cap B')$ ....(i) **1**

Now R.H.S. take

$y \in (A' \cap B')$

$\Rightarrow \quad y \in A'$ and $y \in B'$ **½**

$\Rightarrow \quad y \notin A$ and $y \notin B$

$\Rightarrow \quad y \notin (A \cup B)$

$\Rightarrow \quad y \in (A \cup B)'$ **½**

Thus $\quad y \in (A' \cap B') \Rightarrow y \in (A \cup B)'$

$\Rightarrow \quad (A' \cap B') \subset (A \cup B)'$ ....(ii)

From equations (i) and (ii),

$(A \cup B)' = A' \cap B'$ **Hence proved. 1**

8. $\quad f(x) = \sqrt{16 - x^2},\ x \in R$

Domain : $f(x)$ is defined when

$16 - x^2 \geq 0$ **½**

$\Rightarrow \quad x^2 - 16 \leq 0$

$\Rightarrow \quad (x - 4)(x + 4) \leq 0$ **½**

$\Rightarrow \quad -4 \leq x \leq 4$

$\therefore$ Domain of $f = [-4, 4]$ **1**

Range : $\quad f(x) \geq 0,\ \forall\ x \in (-4, 4)$

Let $\quad f(x) = y = \sqrt{16 - x^2}$

$\Rightarrow \quad y^2 = 16 - x^2$

$\Rightarrow \quad x^2 = 16 - y^2$

$\Rightarrow \quad x = \sqrt{16 - y^2}$ **1**

It is defined, if $\quad 16 - y^2 \geq 0$

$\Rightarrow \quad y^2 - 16 \leq 0$

$\Rightarrow \quad -4 < y \leq 4$

But $\quad y \geq 0$

$\therefore \quad 0 \leq y \leq 4$ **1**

$\therefore \quad$ Range = [0, 4].

9. $\quad$ L.H.S. $= \cos 6x = 2\cos^2 3x - 1,$ $\quad (\because \cos 2x = 2\cos^2 x - 1)$ **1**

$= 2[4\cos^3 x - 3\cos x]^2 - 1$ **1**

$= 2[16\cos^6 x + 9\cos^2 x - 24\cos^4 x] - 1$

$= 32\cos^6 x + 18\cos^2 x - 48\cos^4 x - 1$ **1**

= R.H.S. **Hence proved. 1**

OR

Take $\quad x = \frac{\pi}{8} \Rightarrow 2x = \frac{\pi}{4}$ **½**

$\Rightarrow \quad \tan 2x = \tan \frac{\pi}{4}$

$\Rightarrow \quad \dfrac{2\tan x}{1-\tan^2 x} = 1$ **½**

Let $\quad \tan x = y$

$\Rightarrow \quad 2y = 1 - y^2$

$\Rightarrow \quad y^2 + 2y - 1 = 0$

$\Rightarrow \quad y = \dfrac{-2 \pm \sqrt{4+4}}{2}$ **1**

$\Rightarrow \quad y = \dfrac{-2 \pm 2\sqrt{2}}{2}$

$\Rightarrow \quad y = -1 \pm \sqrt{2}$ **1**

$\Rightarrow \quad \tan x = -1 \pm \sqrt{2}$

$\Rightarrow \quad \tan \dfrac{\pi}{8} = -1 \pm \sqrt{2}$

But $\quad \dfrac{\pi}{8} \in \left(0, \dfrac{\pi}{2}\right)$ **1**

$\therefore \quad \tan \dfrac{\pi}{8} > 0$

$\Rightarrow \quad \boxed{\tan \dfrac{\pi}{8} = -1 + \sqrt{2}}$ **½**

10. A man has a board of length 91 cm.

Let length of smallest piece = x cm

$\therefore$ Length of second piece = $(x + 3)$

and length third piece = $2x$ **½**

$\therefore$ According to question, $x + x + 3 + 2x \leq 91$

$\Rightarrow \quad 3 + 4x \leq 91$ **1**

$\Rightarrow \quad 4x \leq 88$ **1**

$\Rightarrow \quad x \leq 22$ cm.

Also $\quad 2x \geq x + 3 + 5$ **1**

$\Rightarrow \quad x \geq 8.$

Hence, $\quad 8 \leq x \leq 22$

$\therefore$ Smallest piece of board is more than or equal to 8 cm and less than or equal to 22 cm. **½**

11.

$$z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$

$$= \frac{i-1}{\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)}$$ **½**

$$= \frac{2(i-1)}{(1+i\sqrt{3})} \times \frac{(1-i\sqrt{3})}{(1-i\sqrt{3})}$$

$$= \frac{2[i - i^2\sqrt{3} - 1 + i\sqrt{3}]}{1^2 - (i\sqrt{3})^2}$$ **1**

$$= \frac{2[(\sqrt{3}-1) + i(\sqrt{3}+1)]}{4}$$

$$z = \left(\frac{\sqrt{3}-1}{2}\right) + i\left(\frac{\sqrt{3}+1}{2}\right)$$

Let $\frac{\sqrt{3}-1}{2} = r\cos\theta$

and $\frac{\sqrt{3}+1}{2} = r\sin\theta$ **½**

$\Rightarrow$ Squaring and adding,

$$r^2 = \frac{(\sqrt{3}-1)^2}{4} + \frac{(\sqrt{3}+1)^2}{4}$$

$$= \frac{1}{4}[3+1+3+1]$$

$$r^2 = 2$$ **1**

$\Rightarrow$ $r = \sqrt{2}$

and $\tan\theta = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$$\tan\theta = \frac{\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\left(1-\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{6}}{1-\tan\frac{\pi}{4}.\tan\frac{\pi}{6}}$$

$$\tan\theta = \tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$$

$\Rightarrow$ $\theta = \frac{\pi}{4}+\frac{\pi}{6} = \frac{5\pi}{12}$ **1**

$\therefore$ $z = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{2}\right)$

This is required polar form.

OR

As $\left(\frac{1+i}{1-i}\right)^m = 1,$ (given)

$\Rightarrow$ $\left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^m = 1$ **1**

$\Rightarrow$ $\left[\frac{(1+i)^2}{1-i^2}\right]^m = 1$

$\Rightarrow$ $\left[\frac{1+i^2+2i}{2}\right]^m = 1$ **1**

$\Rightarrow$ $\left(\frac{1-1+2i}{2}\right)^m = 1$ **1**

$\Rightarrow \qquad i^m = 1$

It is possible when least value of m is 4. **1**

$\because \qquad i^4 = 1.$

12. We have a word *INDEPENDENCE* consists 12 letters with $N \to 3$, $D \to 2$, $E \to 4$ and others are different.

(i) There are 5 vowels in it with $E \to 4$ and I, they should occur together *i.e.,* $\boxed{EEEEI}$ as a single word and remaining 7. **1**

Letters are separate.

$$\therefore \quad \text{Total such arrangements} = \frac{8! \times 5!}{2! \times 3! \times 4!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4! \times 5 \times 4 \times 3}{2 \times 3! \times 4!} \qquad \textbf{½}$$

$$= 16800. \qquad \textbf{½}$$

(ii) Word begin with I and end with P

i.e., $I \boxed{NDEENDENCE} P$ **½**

So, we have arrange only 10 letters. **½**

$$\therefore \quad \text{Total such arrangements} = \frac{10!}{4! \times 3! \times 2!} \qquad \textbf{½}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 3 \times 2 \times 2}$$

$$= \frac{720 \times 35}{2} = 360 \times 35 \qquad \textbf{½}$$

$$= 12600.$$

OR

We need to use digits 0, 1, 2, 2, 2, 4, 4, to obtain a number more than 1000000.

It is 7 digit number and we also have 7 digits so all digits should be used once. **1**

$$\text{Total 7 digits numbers using these digits} = \frac{7!}{3! \times 2!} = \frac{7 \times 6 \times 5 \times 4 \times 3!}{3! \times 2 \times 1}$$

$$= 420 \qquad \textbf{1}$$

But it includes numbers that also start with zero.

$$\text{So,} \quad \text{number that start with zero} = \frac{6!}{2! \times 3!} = \frac{6 \times 5 \times 4 \times 3!}{2 \times 3!} = 60 \qquad \textbf{1}$$

$\therefore$ Total 7 digits numbers more than 1000000

$$= 420 - 60 = 360. \qquad \textbf{1}$$

13. $$P(n) : (1 + x)^n \geq 1 + nx, \; x \in N \text{ and } x > -1.$$

Put $n = 1$

$$P(1) : (1 + x) > 1 + x \quad \text{(True)}$$

So, $P(1)$ in true.

Let $P(k)$ is true

i.e., $$(1 + x)^k \geq 1 + kx \qquad \text{....(i)} \quad \textbf{½}$$

Now to prove $P(k + 1)$ is true.

i.e., $(1 + x)^{k+1} \geq 1 + (k + 1)x$ ....(ii) **½**

Taking L.H.S.

$$(1 + x)^{k+1} = (1 + x)^k \cdot (1 + x)$$

Using equation (i), **1**

$$(1 + x)^{k+1} \geq (1 + kx)(1 + x)$$

$$\geq kx^2 + kx + x + 1$$

As $x^2 \geq 0 \Rightarrow kx^2 \geq 0$ **1**

$\Rightarrow$ $(1 + x)^{k+1} \geq (kx + x + 1)$

$(1 + x)^{k+1} \geq [1 + (k + 1)x]$ **1**

i.e., $P(k + 1)$ is true. Hence $P(n)$ is true for all $n \in N$.

14. Let a and b are two numbers then A.M. = $A = \dfrac{a+b}{2}$ and G.M. = $\sqrt{ab} = G$ **1**

Let a and b are roots of quadratic equation, then

$x^2 - (a + b)x + ab = 0$ **1**

But $a + b = 2A$, and $ab = G^2$

$\Rightarrow$ $x^2 - 2Ax + G^2 = 0$

Using quadratic formula, we get

$\therefore$ $x = \dfrac{2A \pm \sqrt{4A^2 - 4G^2}}{2}$ **1**

$= \dfrac{2A \pm 2\sqrt{A^2 - G^2}}{2}$

$\Rightarrow$ $x = A \pm \sqrt{(A+G)(A-G)}$ **1**

i.e., $a = A + \sqrt{(A+G)(A-G)}$

and $b = A - \sqrt{(A+G)(A-G)}$ **Hence proved.**

15. There are 25 students, 10 are to be chosen, 3 students decided to join all of them or none of them. **1**

So, when all these 3 students are joining them we need to select 7 students out of 22.

i.e., in ${}^{22}C_7$ ways **1**

and when these 3 dicided not to go, then select 10 students out of 22.

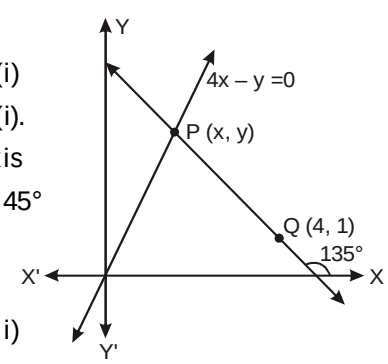
i.e., in ${}^{22}C_{10}$ ways **1**

So, total ways of selection of 10 students

$= {}^{22}C_7 + {}^{22}C_{10}$. **1**

16. Equation of line is

$4x - y = 0$ ....(i)



It is passing through origin, we need to find distance of line (i).

From Q (4, 1) along a line, makes angle 135° with + ve x-axis

$\therefore$ Slope of PQ = tan 135° = tan (180 – 45°) = – tan 45°

$= -1$ **1**

$\therefore$ Equation of PQ, $y - 1 = -1(x - 4)$ **1**

$\Rightarrow$ $x + y = 5$ ....(ii)

On solving (i) and (ii), we get

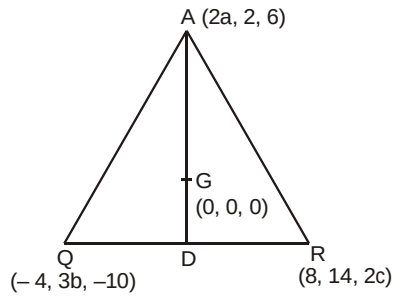
$$n = 1,\ y = 4 \text{ i.e., } P\ (1, 4)$$ **1**

$\therefore$ $$PQ = \sqrt{(4-1)^2 + (1-4)^2} = \sqrt{9+9}$$

$$PQ = 3\sqrt{2} \text{ units.}$$

i.e., Distance of line (i) from (4, 1) = $3\sqrt{2}$ units. **1**

17. The vertices of triangle *PQR* are *P* (2*a*, 2, 6), *Q* (– 4, 3*b*, – 10) and *R* (8, 14, 2*c*) and centroid is at orign.



$\therefore$ $$\frac{2a-4+8}{3} = 0$$

$\Rightarrow$ $$2a + 4 = 0 \Rightarrow a = -2$$ **1**

$$\frac{2+3b+14}{3} = 0$$

$\Rightarrow$ $$3b + 16 = 0 \Rightarrow b = -16/3$$ **1**

and $$\frac{6-10+2c}{3} = 0$$

$\Rightarrow$ $$-4 + 2c = 0$$

$\Rightarrow$ $$c = 2$$ **1**

18. $\lim\limits_{x\to 0}\left(\frac{(x+1)^5-1}{x}\right)$; $\left(\frac{0}{0} \text{ form}\right)$

$$= \lim_{x\to 0}\left[\frac{(x+1)^5-(1)^5}{(x+1)-1}\right]$$ **1**

Let $x + 1 = y \Rightarrow y \to 1$ **1**

$$= \lim_{y\to 1}\left[\frac{y^5-1^5}{y-1}\right] \qquad \left[\because \lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right) = na^{n-1}\right]$$ **1**

$\Rightarrow$ $$5\ (1)^{5-1} = 5.$$ **1**

OR

$\lim\limits_{x\to \pi}\left(\frac{\sin(\pi - x)}{\pi(\pi - x)}\right)$; $\left(\frac{0}{0} \text{ form}\right)$ **½**

Apply R.H.L., put $x = \pi + h$, $h \to 0$ **½**

$$= \lim_{h\to 0}\left[\frac{\sin(\pi-\pi-h)}{\pi(\pi-\pi-h)}\right]$$ **1**

$$= \lim_{h\to 0}\left[\frac{\sin(-h)}{\pi(-h)}\right]$$ **1**

$$= \frac{1}{\pi}\lim_{h\to 0}\left[\frac{\sin h}{h}\right]$$ **1**

$$= \frac{1}{\pi}, \qquad \left(\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right)$$

19. $z_1 = 2 - i$ and $z_2 = 1 + i$

$$\therefore \quad \left(\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right) = \left(\frac{2 - i + 1 + i + 1}{2 - i - 1 - i + i}\right) \qquad \textbf{1}$$

$$= \left(\frac{4}{1 - i}\right)$$

$$= \left(\frac{4(1 + i)}{(1 + i)(1 - i)}\right) \qquad \textbf{1}$$

$$= \left(\frac{4(1 + i)}{1 - i^2}\right)$$

$$= 2\,(1 + i) \qquad \textbf{1}$$

$$\therefore \quad \left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right| = |2(1 + i)|$$

$$= 2\sqrt{1^2 + (1)^2}$$

$$= 2\sqrt{2}. \qquad \textbf{1}$$

Section-C

20. Let, A = Number of students in Chemistry class.

i.e., $n(A) = 40$

and B = No. of students in Physics class. **1**

i.e., $n(B) = 60$

(i) When two classes meet at the same hour than there is no common student in these classes

i.e., $n(A \cap B) = 0$ **1**

$\therefore$ $n(A \cup B) = n(A) + n(B) - n(A \cup B)$

$= 40 + 60 - 0$

$n(A \cup B) = 100$ **1**

(ii) When classes meet at different hour, then there are 20 common students.

i.e., $n(A \cap B) = 20$ **1**

$\therefore$ $n(A \cap B) = 40 + 60 - 20$

$= 100 - 20$

$n(A \cup B) = 20.$ **1**

Parents encourage their wards to these streams because of

(i) Jobs oriented streams.

(ii) Develope a scientific skills.

(iii) Technical education.

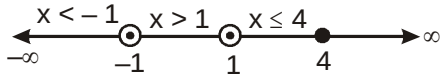
21.

$$f(x) = \sqrt{4 - x} + \frac{1}{\sqrt{x^2 - 1}},\ x \in R$$

For domain : $4 - x \geq 0 \Rightarrow x \leq 4$ **2**

and $x^2 - 1 > 0 \Rightarrow (x - 1)(x + 1) > 0$

$\Rightarrow$ $x < -1$ and $x > 1$ **2**



i.e., domain of $f = (-\infty, -1) \cup (1, 4]$. **2**

OR

$A = \{1, 2, 3, 4, 5, 6\}$

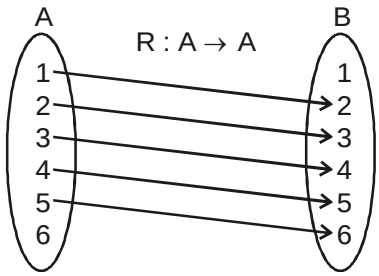
$R = \{(x, y) : y = x + 1; x, y \in A\}$.

i.e., $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$ **2**

$\therefore$ Domain = {1, 2, 3, 4, 5} **1**

Co-domain = {1, 2, 3, 4, 5, 6} **1**

and Range = {2, 3, 4, 5, 6} ½



1½

Arrow diagram

22. There are 10 points in a plane, so straight lines can be formed using two at a time.

$\therefore$ Total straight lines = ${}^{10}C_2 = \dfrac{10 \times 9}{2 \times 1} = 45$ **1**

But 4 points are collinear.

$\therefore$ Lines using these points = ${}^{4}C_2 = \dfrac{4 \times 3}{2 \times 1} = 6$ **1**

But these 4 points can make a single line.

So, total different lines = 45 – 6 + 1 = 40

Now triangle can be formed using 3 points at a time.

So, Number of triangles = ${}^{10}C_3 = \dfrac{10 \times 9 \times 8}{3 \times 2 \times 1}$

= 120. **1**

Using 4 points taking 3 at a time.

No. of triangles = ${}^{4}C_3 = 4$ **1**

But collinear points can't make only tirangle.

So, total no. of triangles = 120 – 4 = 116. **1**

23. Expansion of $\left(x - \dfrac{3}{x^2}\right)^m$

$$= {}^mC_0\, x^m - {}^mC_1\, x^{m-1} \times \frac{3}{x^2} + {}^mC_2 x^{m-2} \times \left(\frac{3}{x^2}\right)^2 + ...$$

$\therefore$ Coefficients of first three terms of this expansion are ${}^mC_0$, $-3.{}^mC_1$ and $9.{}^mC_2$ **½**

According to question,

$${}^mC_0 + (-3).{}^mC_1 + 9.{}^mC_2 = 559$$ **½**

$\Rightarrow$ $1 - 3m + \dfrac{9m(m-1)}{2} = 559$

$\Rightarrow$ $9m^2 - 9m + 2 - 6m = 2 \times 559$ **1**

$\Rightarrow$ $9m^2 - 15m + (2 - 2 \times 559) = 0$

$\Rightarrow$ $9m^2 - 15m - 2 \times 558 = 0$

$\Rightarrow$ $3m^2 - 5m - 372 = 0$

$\Rightarrow$ $3m^2 - 36m + 31m - 372 = 0$

$\Rightarrow$ $3m\,(m - 12) + 31\,(m - 12) = 0$ **1**

$\Rightarrow \quad (m - 12)(3m + 31) = 0$

$\Rightarrow \quad m = -\dfrac{31}{3}$ (Not possible) $\quad (\because m \in N)$

and $\quad m = 12$

$\therefore$ Expansion is now $\left(x - \dfrac{3}{x^2}\right)^{12}$

$\therefore \quad T_{r+1} = (-1)^r \cdot {}^{12}C_r\, (n)^{12-r} \left(\dfrac{3}{x^2}\right)^r$ **1**

$= (-1)^r \cdot {}^{12}C_r\, x^{12-r-2r} .\ 3^r$

For coefficients of x^3, put

$12 - 3r = 3 \Rightarrow r = 3$ **1**

$\therefore$ Term containing, $\quad x^3 = (-1)^3\, {}^{12}C_3\, 3^3 x^3$

$= \dfrac{12 \times 11 \times 10}{3 \times 2 \times 1} \times 3 \times 3 \times 3 \times x^3$

$= -132 \times 45 \times x^3$

$= -5940x^3$ **1**

OR

a and b are distinct integers (given), then

$a^n = (a - b + b)^x$ **1**

Using binomial themorem,

$a^n = [(a-b)^n + {}^nC_1 (a-b)^{n-1} .\ b + {}^nC_2 (a-b)^{n-2} b^2$

$+ {}^nC_3 (a-b)^{n-3} .\ b^3 + ... + {}^nC_n b^n]$ **1**

$a^n - b^n = [(a-b)^n + {}^nC_1 (a-b)^n (a-b)^{-1} b + {}^nC_2 (a-b)^n (a-b)^{-2} b^2 + ...]$ **1**

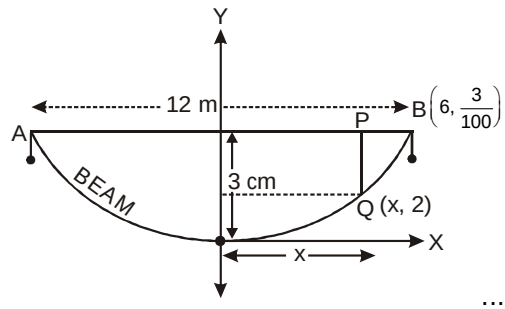
$a^n - b^n = (a-b)\, [(a-b)^{n-1} + {}^nC_1 (a-b)^{n-2} b + {}^nC_2 (a-b)^{n-3} b^2 + ...]$ **1**

Let $\quad \lambda = (a-b)^{n-1} + {}^nC_1 (a-b)^{n-2} b + {}^nC_2 (a-b)^{n-3} b^2 + ...]$ **1**

$\therefore \quad a^n - b^n = \lambda\, (a - b)$

$\therefore$ $(a - b)$ is a factor of $a^n - b^n$. **1**

24. *AOB* is a beam supported at the ends *A* and *B* and deflection is 3 cm at centre and beam is in parabolic shape. **1**



$\therefore$ Equation of beam is $x^2 = 4ay$ **1**

But $B\left(6, \dfrac{3}{100}\right)$ lies on it

i.e., $\quad 36 = 4a \times \dfrac{3}{100}$

$\Rightarrow \quad 4a = 1200$ **1**

$\Rightarrow \quad x^2 = 1200\, y$ ....(i)

Let PQ = 1 cm deflection in beam at x cm distance from centre.

$\therefore$ $Q\left(x, \dfrac{2}{100}\right)$ lies on (i). **1**

$\therefore \quad x^2 = 1200 \times \dfrac{2}{100}$ **1**

$\Rightarrow \quad x^2 = 24$

$\Rightarrow \quad x = \sqrt{24}$

$\Rightarrow$ $x = 2\sqrt{6}$ metres **1**

$\therefore$ 1 cm deflection is at $2\sqrt{6}$ metres distance from centre.

OR

Equation of ellipse,

$$9x^2 + 4y^2 = 36$$

$\Rightarrow$ $\frac{x^2}{4} + \frac{y^2}{9} = 1$ **1**

$\Rightarrow$ $a^2 = 4 \Rightarrow a = \pm 2$ **1**

and $b^2 = 9 \Rightarrow b = \pm 3$

$\therefore$ y-axis is the major axis.

$\therefore$ $c^2 = b^2 - a^2 = 9 - 4 = 5$ **1**

$\Rightarrow$ $c = \pm 5$

Foci = $(0, \pm 5)$ **½**

Vertices = $(0, \pm 3)$ **½**

Length of major axis = $2b = 2 \times 3 = 6$ units **1**

and Eccentricity *i.e.,* $e = \frac{c}{b} = \frac{\sqrt{5}}{3}$ **1**

25. There are 52 playing cards.

One card is to be drawn.

(i) P (Diamond card) = $\frac{13}{52} = \frac{1}{4}$. **1**

(ii) A = Black card

$\therefore$ $n(A) = 26$

$\therefore$ $P(A) = \frac{26}{52} = \frac{1}{2}$. **1**

(iii) B = Not an ace

$\therefore$ $n(B) = 52 - 4 = 48$

$\therefore$ $P(B) = \frac{48}{52} = \frac{12}{13}$ **1**

(iv) C = Not a diamond card

$\therefore$ $n(C) = 52 - 13 = 39$ cards

$\therefore$ $P(C) = \frac{39}{52} = \frac{3}{4}$ **1**

(v) D = Not a black card *i.e.,* red cards

$\therefore$ $n(D) = 26$

$\therefore$ $P(D) = \frac{26}{52} = \frac{1}{2}$ **1**

(vi) E = A face card

$\therefore$ $n(E) = 12$ cards (4; king, queen and jack each)

$\therefore$ $P(E) = \frac{12}{52} = \frac{3}{13}$ **1**

26. There are two firms belonging to the same industry say *A* and *B*.

In firm *A*,

No. of earners = 586

Mean wages of earners = 5253

and variance = 100

∴ Wages (total) paid by firm

= 586 × 5253 = 30,78,258

and S.D. = $\sqrt{\text{variance}} = \sqrt{100} = 10$ **1**

In firm *B*,

No. of wage earners = 648

Mean wages = 5253

and variance = 121

∴ Total wages paid by firm

= 648 × 5253 **1**

= 34,03,944.

S.D. = $\sqrt{\text{variance}} = \sqrt{121} = 11$ **1**

∴ Firm *B* pays larger amount of wages to their workers. Now, to compare variability between two firms we need to find coefficients of variation.

C.V. (Firm *A*) = $\frac{\text{S.D.}}{\text{Mean}} \times 100$ **1**

$= \frac{10}{5253} \times 100 = 0{\cdot}190$

C.V. (Firm *B*) = $\frac{11}{5253} \times 100 = 0{\cdot}2094$ **1**

Hence, firm *B* shows greater variability between firms. It means firm *A* looks more consistent than firm *B* about their wages. **1**

❑❑